Lesson 8: Graphs of Simple Nonlinear Functions

Classwork

Exploratory Challenge/Exercises 1–3

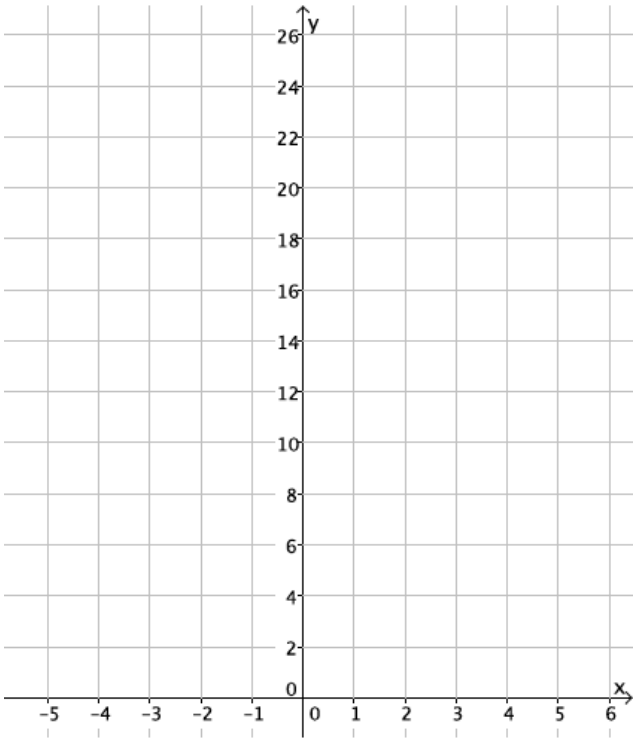
1. Consider the function that assigns to each number x the value x^2.

   a. Do you think the function is linear or nonlinear? Explain.

   b. Develop a list of inputs and outputs for this function. Organize your work using the table below. Then, answer the questions that follow.

Input (x)	Output (x^2)
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

c. Plot the inputs and outputs as ordered pairs defining points on the coordinate plane.



d. What shape does the graph of the points appear to take?

e. Find the rate of change using rows 1 and 2 from the table above.

f. Find the rate of change using rows 2 and 3 from the table above.

g. Find the rate of change using any two other rows from the table above.

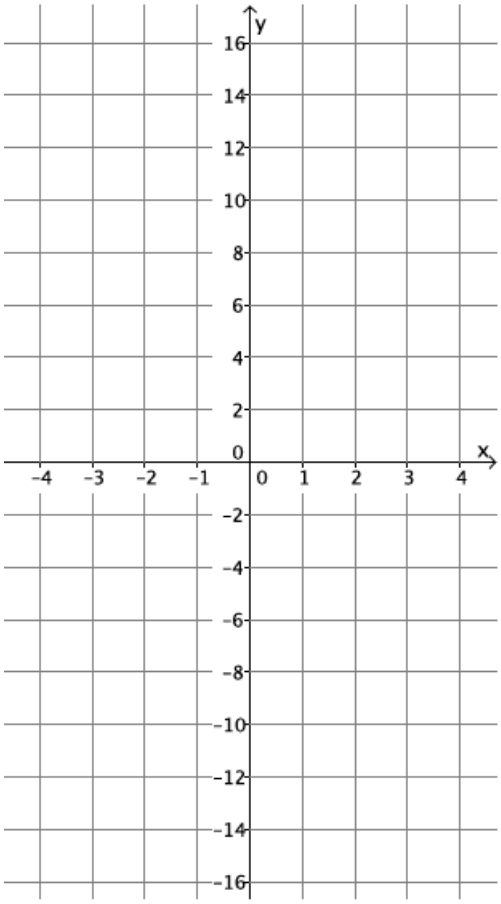
h. Return to your initial claim about the function. Is it linear or nonlinear? Justify your answer with as many pieces of evidence as possible.

2. Consider the function that assigns to a number x the value x^3.

a. Do you think the function is linear or nonlinear? Explain.

b. Develop a list of inputs and outputs for this function. Organize your work using the table below. Then, answer the questions that follow.

Input (x)	Output (x^3)
-2.5	
-2	
-1.5	
-1	
-0.5	
0	
0.5	
1	
1.5	
2	
2.5	



c. Plot the inputs and outputs as ordered pairs defining points on the coordinate plane.

d. What shape does the graph of the points appear to take?

e. Find the rate of change using rows 2 and 3 from the table above.

f. Find the rate of change using rows 3 and 4 from the table above.

g. Find the rate of change using rows 8 and 9 from the table above.

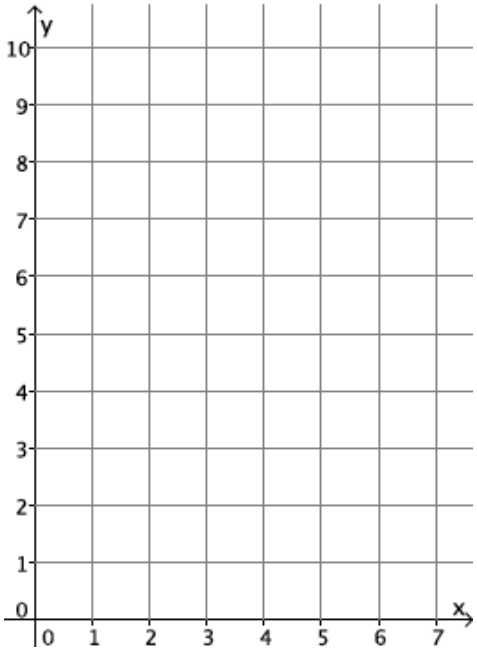
h. Return to your initial claim about the function. Is it linear or nonlinear? Justify your answer with as many pieces of evidence as possible.

3. Consider the function that assigns to each positive number x the value $\frac{1}{x}$.

a. Do you think the function is linear or nonlinear? Explain.

b. Develop a list of inputs and outputs for this function. Organize your work using the table below. Then, answer the questions that follow.

Input (x)	Output $\left(\frac{1}{x}\right)$
0.1	
0.2	
0.4	
0.5	
0.8	
1	
1.6	
2	
2.5	
4	
5	



c. Plot the inputs and outputs as ordered pairs defining points on the coordinate plane..

d. What shape does the graph of the points appear to take?

e. Find the rate of change using rows 1 and 2 from the table above.

f. Find the rate of change using rows 2 and 3 from the table above.

g. Find the rate of change using any two other rows from the table above.

h. Return to your initial claim about the function. Is it linear or nonlinear? Justify your answer with as many pieces of evidence as possible.

Exercises 4–10

In each of Exercises 4–10, an equation describing a rule for a function is given, and a question is asked about it. If necessary, use a table to organize pairs of inputs and outputs, and then plot each on a coordinate plane to help answer the question.

4. What shape do you expect the graph of the function described by $y = x$ to take? Is it a linear or nonlinear function?

5. What shape do you expect the graph of the function described by $y = 2x^2 - x$ to take? Is it a linear or nonlinear function?

6. What shape do you expect the graph of the function described by $3x + 7y = 8$ to take? Is it a linear or nonlinear function?

7. What shape do you expect the graph of the function described by $y = 4x^3$ to take? Is it a linear or nonlinear function?

8. What shape do you expect the graph of the function described by $\frac{3}{x} = y$ to take? Is it a linear or nonlinear function? (Assume that an input of $x = 0$ is disallowed.)

9. What shape do you expect the graph ofthe function described by $\frac{4}{x^2} = y$ to take? Is it a linear or nonlinear function? (Assume that an input of $x = 0$ is disallowed.)

10. What shape do you expect the graph of the equation $x^2 + y^2 = 36$ to take? Is it a linear or nonlinear function? Is it a function? Explain.

Lesson Summary

One way to determine if a function is linear or nonlinear is to inspect average rates of change using a table of values. If these average rates of change are not constant, then the function is not linear.

Another way is to examine the graph of the function. If all the points on the graph do not lie on a common line, then the function is not linear.

If a function is described by an equation different from one equivalent to $y = mx + b$ for some fixed values m and b, then the function is not linear.

Problem Set

1. Consider the function that assigns to each number x the value $x^2 - 4$.

   a. Do you think the function is linear or nonlinear? Explain.

   b. Do you expect the graph of this function to be a straight line?

   c. Develop a list of inputs and matching outputs for this function. Use them to begin a graph of the function.

   d. Was your prediction to (b) correct?

Input (x)	Output $(x^2 - 4)$
-3	
-2	
-1	
0	
1	
2	
3	

2. Consider the function that assigns to each number x greater than -3 the value $\frac{1}{x+3}$.

   a. Is the function linear or nonlinear? Explain.

   b. Do you expect the graph of this function to be a straight line?

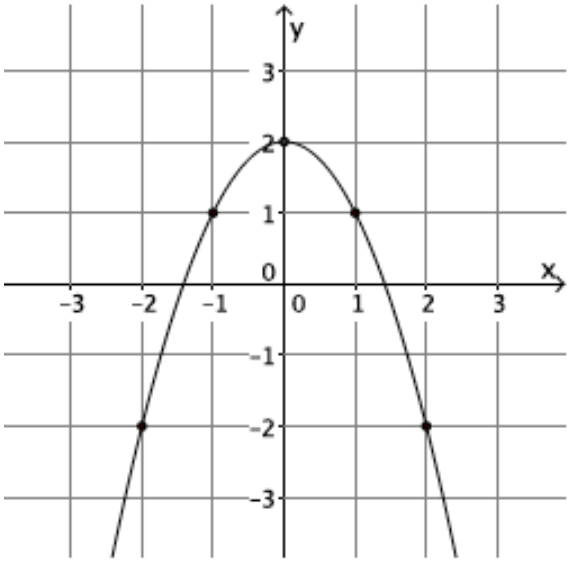
   c. Develop a list of inputs and matching outputs for this function. Use them to begin a graph of the function.

   d. Was your prediction to (b) correct?

Input (x)	Output $\left(\frac{1}{x+3}\right)$
-2	
-1	
0	
1	
2	
3	

3.

a. Is the function represented by this graph linear or nonlinear? Briefly justify your answer.



b. What is the average rate of change for this function from an input of $x = -2$ to an input of $x = -1$?

c. What is the average rate of change for this function from an input of $x = -1$ to an input of $x = 0$?