Lesson 3: Linear Functions and Proportionality

Classwork

Example 1

In the last lesson, we looked at several tables of values showing the inputs and outputs of functions. For instance, one table showed the costs of purchasing different numbers of bags of candy:

Bags of candy (x)	1	2	3	4	5	6	7	8
Cost in Dollars (y)	1.25	2.50	3.75	5.00	6.25	7.50	8.75	10.00

Example 2

Walter walks at a constant speed of 8 miles every 2 hours. Describe a linear function for the number of miles he walks in x hours. What is a reasonable range of x-values for this function?

Example 3

Veronica runs at a constant speed. The distance she runs is a function of the time she spends running. The function has the table of values shown below.

Time in minutes (x)	8	16	24	32
Distance run in miles (y)	1	2	3	4

Example 4

Water flows from a faucet into a bathtub at the constant rate of 7 gallons of water pouring out every 2 minutes. The bathtub is initially empty, and its plug is in. Determine the rule that describes the volume of water in the tub as a function of time. If the tub can hold 50 gallons of water, how long will it take to fill the tub?

Now assume that you are filling the same 50-gallon bathtub with water flowing in at the constant rate of 3.5 gallons per minute, but there were initially 8 gallons of water in the tub. Will it still take about 14 minutes to fill the tub?

Time in minutes (x)	0	3	6	9	12
Total volume in tub in gallons (y)					

Example 5

Water flows from a faucet at a constant rate. Assume that $\mathbf{6}$ gallons of water are already in a tub by the time we notice the faucet is on. This information is recorded in the first column of the table below. The other columns show how many gallons of water are in the tub at different numbers of minutes since we noticed the running faucet.

Time in minutes (x)	0	3	5	9
Total volume in tub in gallons (y)	6	9.6	12	16.8

Exercises 1–3

1. Hana claims she mows lawns at a constant rate. The table below shows the area of lawn she can mow over different time periods.

Number of minutes (x)	5	20	30	50
Area mowed in square feet (y)	36	144	216	360

   a. Is the data presented consistent with the claim that the area mowed is a linear function of time?

b. Describe in words the function in terms of area mowed and time.

c. At what rate does Hana mow lawns over a 5-minute period?

d. At what rate does Hana mow lawns over a 20-minute period?

e. At what rate does Hana mow lawns over a 30-minute period?

f. At what rate does Hana mow lawns over a 50-minute period?

g. Write the equation that describes the area mowed, y, in square feet, as a linear function of time, x, in minutes.

h. Describe any limitations on the possible values of x and y.

i. What number does the function assign to $x = 24$? That is, what area of lawn can be mowed in 24 minutes?

j. According to this work, how many minutes would it take to mow an area of 400 square feet?

2. A linear function has the table of values below. The information in the table shows the total volume of water, in gallons, that flows from a hose as a function of time, the number of minutes the hose has been running.

Time in minutes (x)	10	25	50	70
Total volume of water in gallons (y)	44	110	220	308

a. Describe the function in terms of volume and time.

b. Write the rule for the volume of water in gallons, y, as a linear function of time, x, given in minutes.

c. What number does the function assign to 250? That is, how many gallons of water flow from the hose during a period of 250 minutes?

d. The average swimming pool holds about $17{,}300$ gallons of water. Suppose such a pool has already been filled one quarter of its volume. Write an equation that describes the volume of water in the pool if, at time 0 minutes, we use the hose described above to start filling the pool.

e. Approximately how many hours will it take to finish filling the pool?

3. Recall that a linear function can be described by a rule in the form of $y = mx + b$, where m and b are constants. A particular linear function has the table of values below.

Input (x)	0	4	10	11	15	20	23
Output (y)	4	24	54	59			

a. What is the equation that describes the function?

b. Complete the table using the rule.

Lesson Summary

A linear equation $y = mx + b$ describes a rule for a function. We call any function defined by a linear equation a linear function.

Problems involving a constant rate of change or a proportional relationship can be described by linear functions.

Problem Set

1. A food bank distributes cans of vegetables every Saturday. The following table shows the total number of cans they have distributed since the beginning of the year. Assume that this total is a linear function of the number of weeks that have passed.

Number of weeks (x)	1	12	20	45
Total number of cans of vegetables distributed (y)	180	2,160	3,600	8,100

   a. Describe the function being considered in words.

   b. Write the linear equation that describes the total number of cans handed out, y, in terms of the number of weeks, x, that have passed.

   c. Assume that the food bank wants to distribute $20{,}000$ cans of vegetables. How long will it take them to meet that goal?

   d. The manager had forgotten to record that they had distributed $35{,}000$ cans on January 1. Write an adjusted linear equation to reflect this forgotten information.

   e. Using your function in part (d), determine how long in years it will take the food bank to hand out $80{,}000$ cans of vegetables.

2. A linear function has the table of values below. It gives the number of miles a plane travels over a given number of hours while flying at a constant speed.

Number of hours traveled (x)	2.5	4	4.2
Distance in miles (y)	1,062.5	1,700	1,785

   a. Describe in words the function given in this problem.

   b. Write the equation that gives the distance traveled, y, in miles, as a linear function of the number of hours, x, spent flying. .

   c. Assume that the airplane is making a trip from New York to Los Angeles, which is a journey of approximately $2{,}475$ miles. How long will it take the airplane to get to Los Angeles?

   d. If the airplane flies for 8 hours, how many miles will it cover?

3. A linear function has the table of values below. It gives the number of miles a car travels over a given number of hours.

Number of hours traveled (x)	3.5	3.75	4	4.25
Distance in miles (y)	203	217.5	232	246.5

   a. Describe in words the function given.

   b. Write the equation that gives the distance traveled, in miles, as a linear function of the number of hours spent driving.

   c. Assume that the person driving the car is going on a road trip to reach a location 500 miles from her starting point. How long will it take the person to get to the destination?

4. A particular linear function has the table of values below.

Input (x)	2	3	8	11	15	20	23
Output (y)	7	10		34		61	

   a. What is the equation that describes the function?

   b. Complete the table using the rule.

5. A particular linear function has the table of values below.

Input (x)	0	5	8	13	15	18	21
Output (y)	6	11	14		21		

   a. What is the rule that describes the function?

   b. Complete the table using the rule.