## Lesson 2: Formal Definition of a Function

## Classwork

## Exercises 1-5

1. Let $D$ be the distance traveled in time $t$. Use the equation $D=16 t^{2}$ to calculate the distance the stone dropped for the given time $t$.

| Time in seconds | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Distance stone fell in feet <br> by that time |  |  |  |  |  |  |  |  |

a. Are the distances you calculated equal to the table from Lesson 1?
b. Does the function $D=16 t^{2}$ accurately represent the distance the stone fell after a given time $t$ ? In other words, does the function described by this rule assign to $t$ the correct distance? Explain.
2. Can the table shown below represent values of a function? Explain.

| Input <br> $(\boldsymbol{x})$ | 1 | 3 | 5 | 5 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Output <br> $(\boldsymbol{y})$ | 7 | 16 | 19 | 20 | 28 |

3. Can the table shown below represent values of a function? Explain.

| Input <br> $(\boldsymbol{x})$ | 0.5 | 7 | 7 | 12 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Output <br> $(\boldsymbol{y})$ | 1 | 15 | 10 | 23 | 30 |

4. Can the table shown below represent values of a function? Explain.

| Input <br> $(\boldsymbol{x})$ | 10 | 20 | 50 | 75 | 90 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Output <br> $(\boldsymbol{y})$ | 32 | 32 | 156 | 240 | 288 |

5. It takes Josephine 34 minutes to complete her homework assignment of 10 problems. If we assume that she works at a constant rate, we can describe the situation using a function.
a. Predict how many problems Josephine can complete in 25 minutes.
b. Write the two-variable linear equation that represents Josephine's constant rate of work.
c. Use the equation you wrote in part (b) as the formula for the function to complete the table below. Round your answers to the hundredths place.

| Time taken to complete <br> problems <br> $(\boldsymbol{x})$ | 5 | 10 | 15 | 20 | 25 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Number of problems completed <br> $(\boldsymbol{y})$ | 1.47 |  |  |  |  |

After 5 minutes, Josephine was able to complete 1.47 problems, which means that she was able to complete 1 problem, then get about halfway through the next problem.
d. Compare your prediction from part (a) to the number you found in the table above.
e. Use the formula from part (b) to compute the number of problems completed when $x=-7$. Does your answer make sense? Explain.
f. For this problem, we assumed that Josephine worked at a constant rate. Do you think that is a reasonable assumption for this situation? Explain.

## Lesson Summary

A function is a correspondence between a set (whose elements are called inputs) and another set (whose elements are called outputs) such that each input corresponds to one and only one output.

Sometimes the phrase exactly one output is used instead of one and only one output in the definition of function (they mean the same thing). Either way, it is this fact, that there is one and only one output for each input, which makes functions predictive when modeling real life situations.

Furthermore, the correspondence in a function is often given by a rule (or formula). For example, the output is equal to the number found by substituting an input number into the variable of a one-variable expression and evaluating.

Functions are sometimes described as an input-output machine. For example, given a function $D$, the input is time $t$, and the output is the distance traveled in $t$ seconds.


## Problem Set

1. The table below represents the number of minutes Francisco spends at the gym each day for a week. Does the data shown below represent values of a function? Explain.

| Day <br> $(\boldsymbol{x})$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time in minutes <br> $(\boldsymbol{y})$ | 35 | 45 | 30 | 45 | 35 | 0 | 0 |

2. Can the table shown below represent values of a function? Explain.

| Input <br> $(\boldsymbol{x})$ | 9 | 8 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Output <br> $(\boldsymbol{y})$ | 11 | 15 | 19 | 24 | 28 |

3. Olivia examined the table of values shown below and stated that a possible rule to describe this function could be $y=-2 x+9$. Is she correct? Explain.

| Input <br> $(x)$ | -4 | 0 | 4 | 8 | 12 | 16 | 20 | 24 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Output <br> $(y)$ | 17 | 9 | 1 | -7 | -15 | -23 | -31 | -39 |

4. Peter said that the set of data in part (a) describes a function, but the set of data in part (b) does not. Do you agree? Explain why or why not.
a.

| Input <br> $(\boldsymbol{x})$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Output <br> $(\boldsymbol{y})$ | 8 | 10 | 32 | 6 | 10 | 27 | 156 | 4 |

b.

| Input <br> $(\boldsymbol{x})$ | -6 | -15 | -9 | -3 | -2 | -3 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Output <br> $(y)$ | 0 | -6 | 8 | 14 | 1 | 2 | 11 | 41 |

5. A function can be described by the rule $y=x^{2}+4$. Determine the corresponding output for each given input.

| Input <br> $(\boldsymbol{x})$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Output <br> $(\boldsymbol{y})$ |  |  |  |  |  |  |  |  |

6. Examine the data in the table below. The inputs and outputs represent a situation where constant rate can be assumed. Determine the rule that describes the function.

| Input <br> $(\boldsymbol{x})$ | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Output <br> $(\boldsymbol{y})$ | 3 | 8 | 13 | 18 | 23 | 28 | 33 | 38 |

7. Examine the data in the table below. The inputs represent the number of bags of candy purchased, and the outputs represent the cost. Determine the cost of one bag of candy, assuming the price per bag is the same no matter how much candy is purchased. Then, complete the table.

| Bags of <br> candy <br> $(\boldsymbol{x})$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cost in <br> Dollars <br> $(\boldsymbol{y})$ |  |  |  | 5.00 | 6.25 |  |  | 10.00 |

a. Write the rule that describes the function.
b. Can you determine the value of the output for an input of $x=-4$ ? If so, what is it?
c. Does an input of -4 make sense in this situation? Explain.
8. Each and every day a local grocery store sells 2 pounds of bananas for $\$ 1.00$. Can the cost of 2 pounds of bananas be represented as a function of the day of the week? Explain.
9. Write a brief explanation to a classmate who was absent today about why the table in part (a) is a function and the table in part (b) is not.
a.

| Input <br> $(\boldsymbol{x})$ | -1 | -2 | -3 | -4 | 4 | 3 | 2 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Output <br> $(\boldsymbol{y})$ | 81 | 100 | 320 | 400 | 400 | 320 | 100 | 81 |

b.

| Input <br> $(x)$ | 1 | 6 | -9 | -2 | 1 | -10 | 8 | 14 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Output <br> $(y)$ | 2 | 6 | -47 | -8 | 19 | -2 | 15 | 31 |

