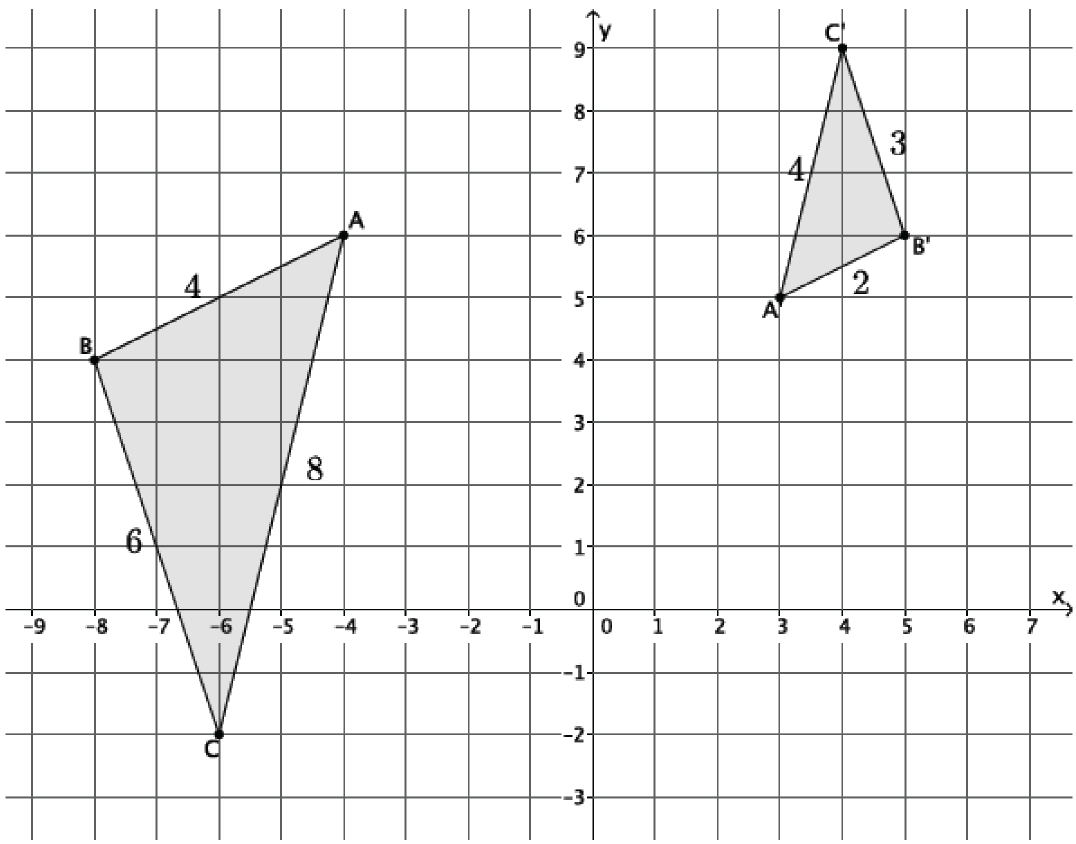
Lesson 9: Basic Properties of Similarity

Classwork

Exploratory Challenge 1

The goal is to show that if $\triangle ABC$ is similar to $\triangle A'B'C'$, then $\triangle A'B'C'$ is similar to $\triangle ABC$. Symbolically, if $\triangle ABC \sim \triangle A'B'C'$, then $\triangle A'B'C' \sim \triangle ABC$.



a. First, determine whether or not $\triangle ABC$ is in fact similar to $\triangle A'B'C'$. (If it isn't, then no further work needs to be done.) Use a protractor to verify that the corresponding angles are congruent and that the ratios of the corresponding sides are equal to some scale factor.

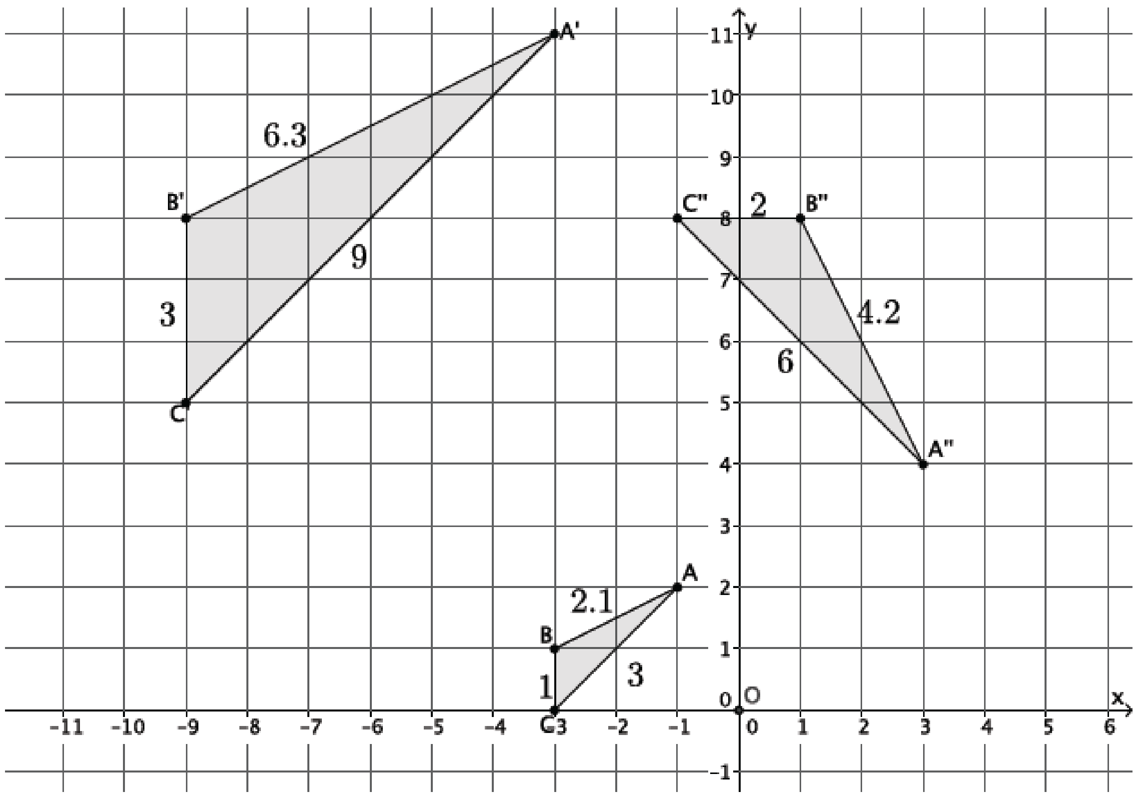
b. Describe the sequence of dilation followed by a congruence that proves $\triangle ABC \sim \triangle A'B'C'$.

c. Describe the sequence of dilation followed by a congruence that proves $\triangle A'B'C' \sim \triangle ABC$.

d. Is it true that $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle ABC$? Why do you think this is so?

Exploratory Challenge 2

The goal is to show that if $\triangle ABC$ is similar to $\triangle A'B'C'$ and $\triangle A'B'C'$ is similar to $\triangle A''B''C''$, then $\triangle ABC$ is similar to $\triangle A''B''C''$. Symbolically, if $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle A''B''C''$, then $\triangle ABC \sim \triangle A''B''C''$.



a. Describe the similarity that proves $\triangle ABC \sim \triangle A'B'C'$.

b. Describe the similarity that proves $\triangle A'B'C' \sim \triangle A''B''C''$.

c. Verify that, in fact, $\triangle ABC \sim \triangle A''B''C''$ by checking corresponding angles and corresponding side lengths. Then, describe the sequence that would prove the similarity $\triangle ABC \sim \triangle A''B''C''$.

d. Is it true that if $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle A''B''\mathrm{C}''$, then $\triangle ABC \sim \triangle A''B''C''$? Why do you think this is so?

Lesson Summary

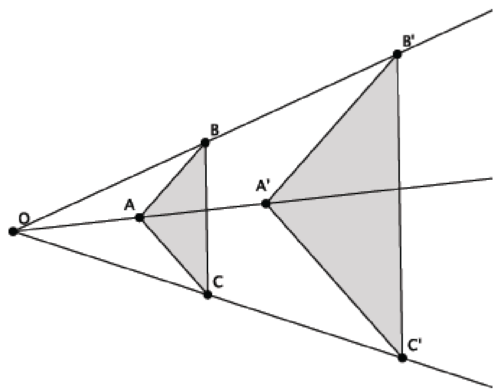
Similarity is a symmetric relation. That means that if one figure is similar to another, $S \sim S'$, then we can be sure that $S' \sim S$.

Similarity is a transitive relation. That means that if we are given two similar figures, $S \sim T$, and another statement about $T \sim U$, then we also know that $S \sim U$.

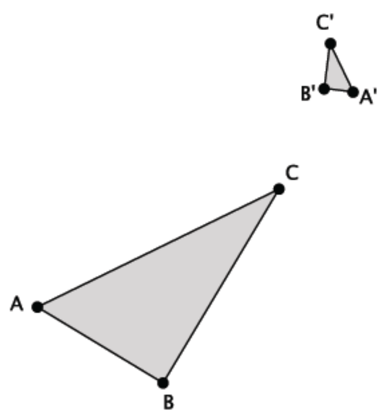
Problem Set

1. Would a dilation alone be enough to show that similarity is symmetric? That is, would a dilation alone prove that if $\triangle ABC \sim \triangle A'B'C'$, then $\triangle A'B'C' \sim \triangle ABC$? Consider the two examples below.

   a. Given $\triangle ABC \sim \triangle A'B'C'$, is a dilation enough to show that $\triangle A'B'C' \sim \triangle ABC$? Explain.



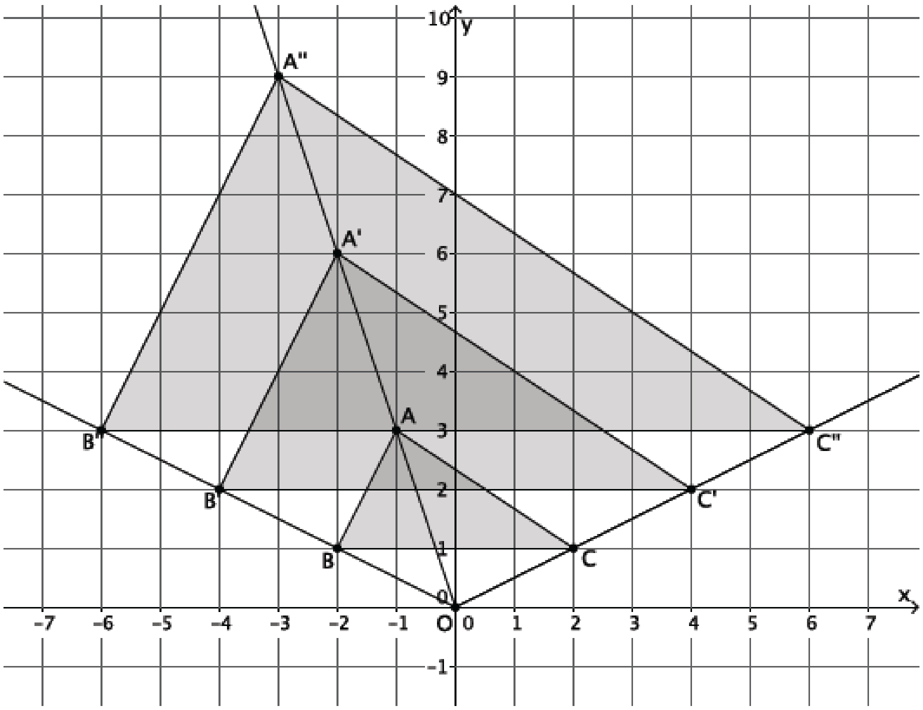
   b. Given $\triangle ABC \sim \triangle A'B'C'$, is a dilation enough to show that $\triangle A'B'C' \sim \triangle ABC$? Explain.



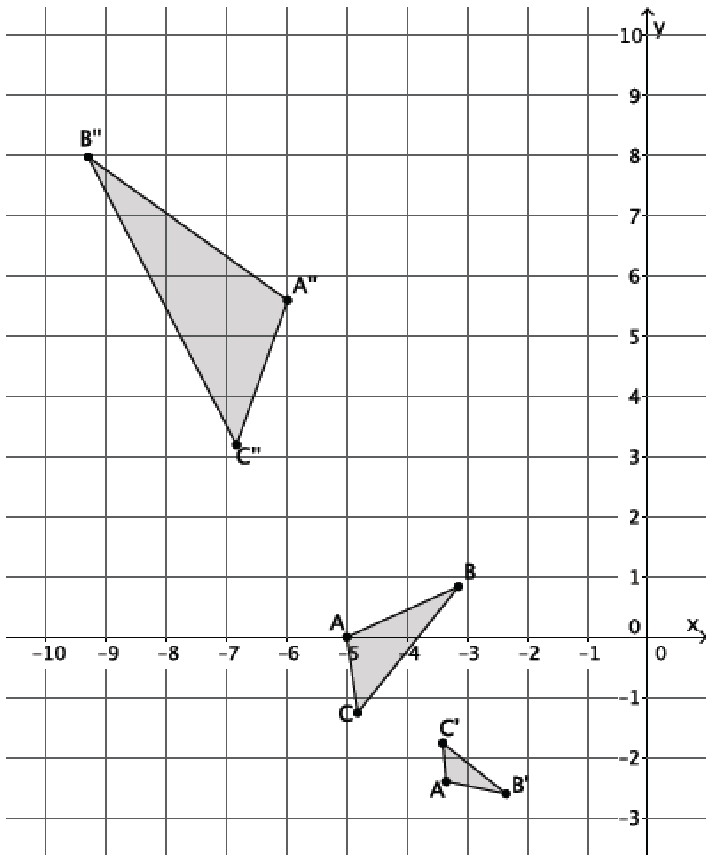
   c. In general, is dilation enough to prove that similarity is a symmetric relation? Explain.

2. Would a dilation alone be enough to show that similarity is transitive? That is, would a dilation alone prove that if $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle A''B''C''$, then $\triangle ABC \sim \triangle A''B''C''$? Consider the two examples below.

   a. Given $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle A''B''C''$, is a dilation enough to show that $\triangle ABC \sim \triangle A''B''C''$? Explain.



b. Given $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle A''B''C''$, is a dilation enough to show that $\triangle ABC \sim \triangle A''B''C''$? Explain.



c. In general, is dilation enough to prove that similarity is a transitive relation? Explain.

3. In the diagram below, $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle A''B''C''$. Is $\triangle ABC \sim \triangle A''B''C''$? If so, describe the dilation followed by the congruence that demonstrates the similarity.