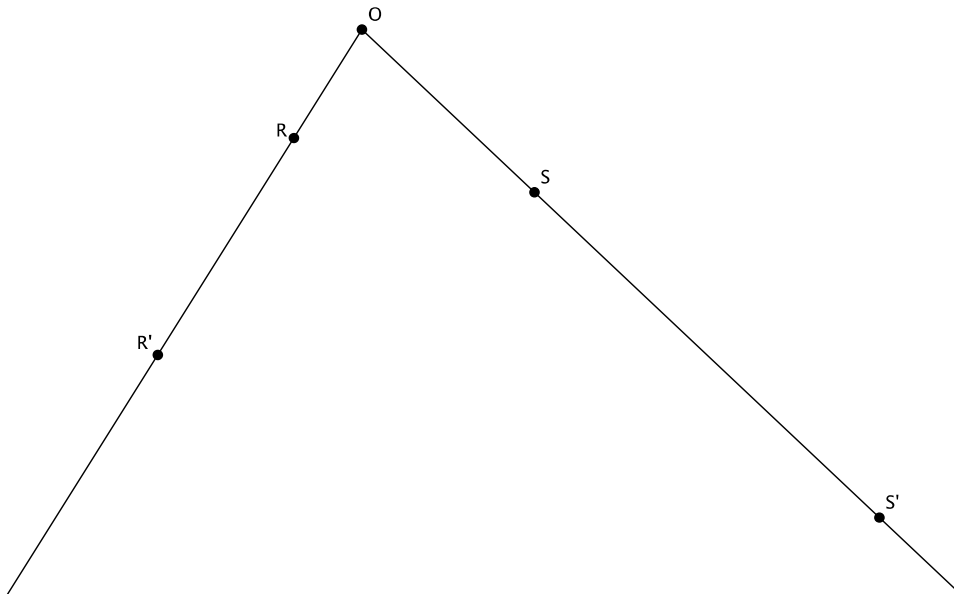


Lesson 4: Fundamental Theorem of Similarity (FTS)

Classwork

Exercise

In the diagram below, points R and S have been dilated from center O by a scale factor of $r = 3$.



a. If $|OR| = 2.3$ cm, what is $|OR'|$?

b. If $|OS| = 3.5$ cm, what is $|OS'|$?

- c. Connect the point R to the point S and the point R' to the point S' . What do you know about the lines that contain segments RS and $R'S'$?
- d. What is the relationship between the length of segment RS and the length of segment $R'S'$?
- e. Identify pairs of angles that are equal in measure. How do you know they are equal?

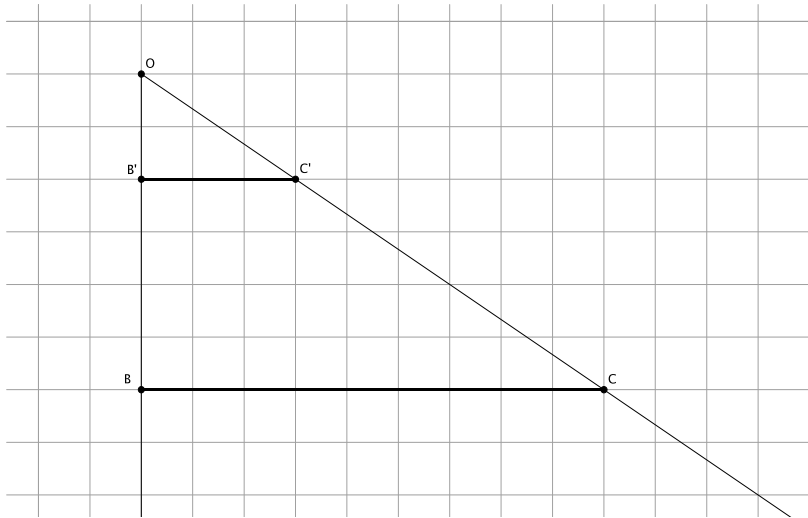
Lesson Summary

THEOREM: Given a dilation with center O and scale factor r , then for any two points P and Q in the plane so that O , P , and Q are not collinear, the lines PQ and $P'Q'$ are parallel, where $P' = \text{Dilation}(P)$ and $Q' = \text{Dilation}(Q)$, and furthermore, $|P'Q'| = r|PQ|$.

Problem Set

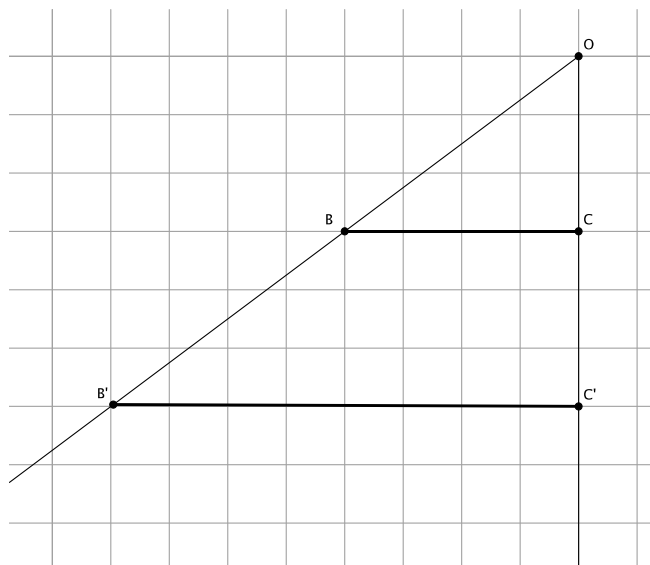
- Use a piece of notebook paper to verify the fundamental theorem of similarity for a scale factor r that is $0 < r < 1$.
 - ✓ Mark a point O on the first line of notebook paper.
 - ✓ Mark the point P on a line several lines down from the center O . Draw a ray, \overrightarrow{OP} . Mark the point P' on the ray and on a line of the notebook paper closer to O than you placed point P . This ensures that you have a scale factor that is $0 < r < 1$. Write your scale factor at the top of the notebook paper.
 - ✓ Draw another ray, \overrightarrow{OQ} , and mark the points Q and Q' according to your scale factor.
 - ✓ Connect points P and Q . Then, connect points P' and Q' .
 - ✓ Place a point, A , on the line containing segment PQ between points P and Q . Draw ray \overrightarrow{OA} . Mark point A' at the intersection of the line containing segment $P'Q'$ and ray \overrightarrow{OA} .
 - Are the lines containing segments PQ and $P'Q'$ parallel lines? How do you know?
 - Which, if any, of the following pairs of angles are equal in measure? Explain.
 - $\angle OPQ$ and $\angle OP'Q'$
 - $\angle OAQ$ and $\angle OA'Q'$
 - $\angle OAP$ and $\angle OA'P'$
 - $\angle OQP$ and $\angle OQ'P'$
 - Which, if any, of the following statements are true? Show your work to verify or dispute each statement.
 - $|OP'| = r|OP|$
 - $|OQ'| = r|OQ|$
 - $|P'A'| = r|PA|$
 - $|A'Q'| = r|AQ|$
 - Do you believe that the fundamental theorem of similarity (FTS) is true even when the scale factor is $0 < r < 1$? Explain.

2. Caleb sketched the following diagram on graph paper. He dilated points B and C from center O .



- What is the scale factor r ? Show your work.
- Verify the scale factor with a different set of segments.
- Which segments are parallel? How do you know?
- Which angles are equal in measure? How do you know?

3. Points B and C were dilated from center O .



- What is the scale factor r ? Show your work.
- If $|OB| = 5$, what is $|OB'|$?
- How does the perimeter of triangle OBC compare to the perimeter of triangle $OB'C'$?
- Did the perimeter of triangle $OB'C' = r \times$ (perimeter of triangle OBC)? Explain.