Lesson 6: Proofs of Laws of Exponents

Classwork

The Laws of Exponents

For $x, y \neq 0$, and all integers a, b, the following holds:

$$x^a \cdot x^b = x^{a+b}$$

$$(x^b)^a = x^{ab}$$

$$(xy)^a = x^a y^a.$$

Facts we will use to prove (11):

(A) (11) is already known to be true when the integers a and b satisfy $a \geq 0$, $b \geq 0$.

(B) $x^{-m} = \dfrac{1}{x^m}$ for any whole number m.

(C) $\left(\dfrac{1}{x}\right)^m = \dfrac{1}{x^m}$ for any whole number m.

Exercise 1

Show that **(C)** is implied by equation (5) of Lesson 4 when $m > 0$, and explain why **(C)** continues to hold even when $m = 0$.
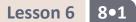
Exercise 2

Show that **(B)** is in fact a special case of (11) by rewriting it as $(x^m)^{-1} = x^{(-1)m}$ for any whole number m, so that if $b = m$ (where m is a whole number) and $a = -1$, (11) becomes **(B)**.

Exercise 3

Show that **(C)** is a special case of (11) by rewriting **(C)** as $(x^{-1})^m = x^{m(-1)}$ for any whole number m. Thus, **(C)** is the special case of (11) when $b = -1$ and $a = m$, where m is a whole number.

Exercise 4

Proof of Case (iii): Show that when $a < 0$ and $b \geq 0$, $(x^b)^a = x^{ab}$ is still valid. Let $a = -c$ for some positive integer c. Show that the left and right sides of $(x^b)^a = x^{ab}$ are equal.

Problem Set

1. You sent a photo of you and your family on vacation to seven Facebook friends. If each of them sends it to five of their friends, and each of those friends sends it to five of their friends, and those friends send it to five more, how many people (not counting yourself) will see your photo? No friend received the photo twice. Express your answer in exponential notation.

# of New People to View Your Photo	Total # of People to View Your Photo

2. Show directly, without using (11), that $(1.27^{-36})^{85} = 1.27^{-36 \cdot 85}$.

3. Show directly that $\left(\frac{2}{13}\right)^{-127} \cdot \left(\frac{2}{13}\right)^{-56} = \left(\frac{2}{13}\right)^{-183}$.

4. Prove for any nonzero number x, $x^{-127} \cdot x^{-56} = x^{-183}$.

5. Prove for any nonzero number x, $x^{-m} \cdot x^{-n} = x^{-m-n}$ for positive integers m and n.

6. Which of the preceding four problems did you find easiest to do? Explain.

7. Use the properties of exponents to write an equivalent expression that is a product of distinct primes, each raised to an integer power.

$$\frac{10^5 \cdot 9^2}{6^4} =$$