# Lesson 13: Analytic Proofs of Theorems Previously Proved by **Synthetic Means**

#### Classwork

## **Opening Exercise**

Let A(30,40), B(60,50), and C(75,120) be vertices of a triangle.

Find the coordinates of the midpoint M of  $\overline{AB}$  and the point  $G_1$  that is the point one-third of the way along  $\overline{MC}$ , closer to M than to C.

b. Find the coordinates of the midpoint N of  $\overline{BC}$  and the point  $G_2$  that is the point one-third of the way along  $\overline{NA}$ , closer to N than to A.

Find the coordinates of the midpoint R of  $\overline{CA}$  and the point  $G_3$  that is the point one-third of the way along  $\overline{RB}$ , closer to R than to B.



engage<sup>ny</sup>

(ce) BY-NC-SA

Lesson 13:

M4

GEOMETHI

### **Exercise 1**

a. Given triangle ABC with vertices  $A(a_1, a_2)$ ,  $B(b_1, b_2)$ , and  $C(c_1, c_2)$ , find the coordinates of the point of concurrency of the medians.

b. Let A(-23, 12), B(13, 36), and C(23, -1) be vertices of a triangle. Where will the medians of this triangle intersect? (Use "Tyler's formula" from part (a) to complete this problem.)



Analytic Proofs of Theorems Previously Proved by Synthetic Means

(cc) BY-NC-SA

Lesson 13:

Lesson 13

M4

**GEOMETRY** 

#### **Exercise 2**

Prove that the diagonals of a parallelogram bisect each other.





Lesson 13:

#### **Problem Set**

- 1. Point M is the midpoint of  $\overline{AC}$ . Find the coordinates of M:
  - a. A(2,3), C(6,10)
  - b. A(-7,5), C(4,-9)
- 2. M(-2,10) is the midpoint of  $\overline{AB}$ . If A has coordinates (4,-5), what are the coordinates of B?
- 3. Line A is the perpendicular bisector of  $\overline{BC}$  with B(-2,-1) and C(4,1).
  - a. What is the midpoint of  $\overline{BC}$ ?
  - b. What is the slope of  $\overline{BC}$ ?
  - c. What is the slope of line A? (Remember, it is perpendicular to  $\overline{BC}$ .)
  - d. Write the equation of line A, the perpendicular bisector of  $\overline{BC}$ .
- 4. Find the coordinates of the intersection of the medians of  $\triangle$  ABC given A(-5,3), B(6,-4), and C(10,10).
- 5. Use coordinates to prove that the diagonals of a parallelogram meet at the intersection of the segments that connect the midpoints of its opposite sides.
- 6. Given a quadrilateral with vertices E(0,5), F(6,5), G(4,0), and H(-2,0):
  - a. Prove quadrilateral *EFGH* is a parallelogram.
  - b. Prove (2, 2.5) is a point on both diagonals of the quadrilateral.
- 7. Prove quadrilateral WXYZ with vertices W(1,3), X(4,8), Y(10,11), and Z(4,1) is a trapezoid.
- 8. Given quadrilateral JKLM with vertices J(-4,2), K(1,5), L(4,0), and M(-1,-3):
  - a. Is it a trapezoid? Explain.
  - b. Is it a parallelogram? Explain.
  - c. Is it a rectangle? Explain.
  - d. Is it a rhombus? Explain.
  - e. Is it a square? Explain.
  - f. Name a point on the diagonal of JKLM. Explain how you know.



engage<sup>ny</sup>

(cc) BY-NC-SA