

Lesson 6: Segments That Meet at Right Angles

Classwork

Opening Exercise

Carlos thinks that the segment having endpoints $A(0,0)$ and $B(6,0)$ is perpendicular to the segment with endpoints $A(0,0)$ and $C(-2,0)$. Do you agree? Why or why not?

Working with a partner, given $A(0,0)$ and $B(3,-2)$, find the coordinates of a point C so that $\overline{AC} \perp \overline{AB}$.

Example

Given points $A(2,2)$, $B(10,16)$, $C(-3,1)$, and $D(4,-3)$, are \overline{AB} and \overline{CD} perpendicular? Are the lines containing the segments perpendicular? Explain.

Exercises

- Given $A(a_1, a_2)$, $B(b_1, b_2)$, $C(c_1, c_2)$, and $D(d_1, d_2)$, find a general formula in terms of a_1 , a_2 , b_1 , b_2 , c_1 , c_2 , d_1 , and d_2 that will let us determine whether \overline{AB} and \overline{CD} are perpendicular.

- Recall the Opening Exercise of Lesson 4 in which a robot is traveling along a linear path given by the equation $y = 3x - 600$. The robot hears a ping from a homing beacon when it reaches the point $F(400, 600)$ and turns to travel along a linear path given by the equation $y - 600 = -\frac{1}{3}(x - 400)$. If the homing beacon lies on the x -axis, what is its exact location? (Use your own graph paper to visualize the scenario.)
 - If point E is the y -intercept of the original equation, what are the coordinates of point E ?

 - What are the endpoints of the original segment of motion?

 - If the beacon lies on the x -axis, what is the y -value of this point, G ?

 - Translate point F to the origin. What are the coordinates of E' , F' , and G' ?

 - Use the formula derived in this lesson to determine the coordinates of point G .

3. A triangle in the coordinate plane has vertices $A(0,10)$, $B(-8,8)$, and $C(-3,5)$. Is it a right triangle? If so, at which vertex is the right angle? (Hint: Plot the points, and draw the triangle on a coordinate plane to help you determine which vertex is the best candidate for the right angle.)
4. $A(-7,1)$, $B(-1,3)$, $C(5,-5)$, and $D(-5,-5)$ are vertices of a quadrilateral. If \overline{AC} bisects \overline{BD} , but \overline{BD} does not bisect \overline{AC} , determine whether $ABCD$ is a kite.

Problem Set

- Are the segments through the origin and the points listed perpendicular? Explain.
 - $A(9,10)$, $B(10,9)$
 - $C(9,6)$, $D(4,-6)$
- Given $M(5,2)$, $N(1,-4)$, and L listed below, are \overline{LM} and \overline{MN} perpendicular? Translate M to the origin, write the coordinates of the images of the points, and then explain without using slope.
 - $L(-1,6)$
 - $L(11,-2)$
 - $L(9,8)$
- Is triangle PQR , where $P(-7,3)$, $Q(-4,7)$, and $R(1,-3)$, a right triangle? If so, which angle is the right angle? Justify your answer.
- A quadrilateral has vertices $(2 + \sqrt{2}, -1)$, $(8 + \sqrt{2}, 3)$, $(6 + \sqrt{2}, 6)$, and $(\sqrt{2}, 2)$. Prove that the quadrilateral is a rectangle.
- Given points $G(-4,1)$, $H(3,2)$, and $I(-2,-3)$, find the x -coordinate of point J with y -coordinate 4 so that the \overrightarrow{GH} and \overrightarrow{IJ} are perpendicular.
- A robot begins at position $(-80,45)$ and moves on a path to $(100,-60)$. It turns 90° counterclockwise.
 - What point with y -coordinate 120 is on this path?
 - Write an equation of the line after the turn.
 - If it stops to charge on the x -axis, what is the location of the charger?
- Determine the missing vertex of a right triangle with vertices $(6,2)$ and $(5,5)$ if the third vertex is on the y -axis. Verify your answer by graphing.
- Determine the missing vertex for a rectangle with vertices $(3,-2)$, $(5,2)$, and $(-1,5)$, and verify by graphing. Then, answer the questions that follow.
 - What is the length of the diagonal?
 - What is a point on both diagonals in the interior of the figure?
- Leg \overline{AB} of right triangle ABC has endpoints $A(1,3)$ and $B(6,-1)$. Point $C(x,y)$ is located in Quadrant IV.
 - Use the perpendicularity criterion to determine at which vertex the right angle is located. Explain your reasoning.
 - Determine the range of values that x is limited to and why.
 - Find the coordinates of point C if they are both integers.