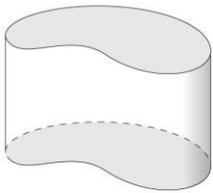# Lesson 7: General Pyramids and Cones and Their Cross-Sections

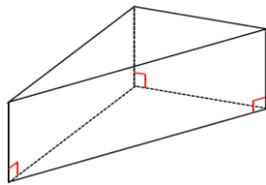## Classwork

### Opening Exercise

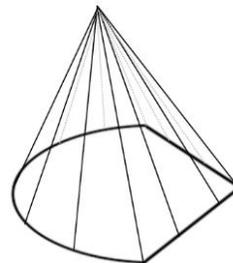Group the following images by shared properties. What defines each of the groups you have made?
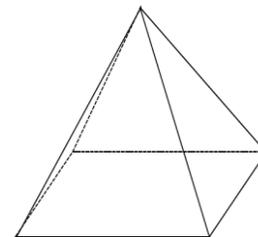


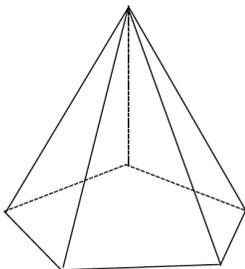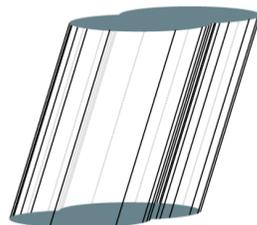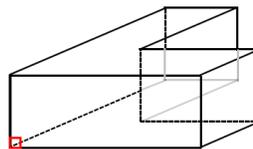| 1 | 2 | 3 | 4 |



| 5 | 6 | 7 | 8 |

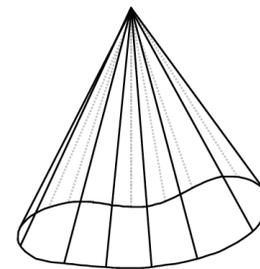**RECTANGULAR PYRAMID:** Given a rectangular region $B$ in a plane $E$ and a point $V$ not in $E$, the *rectangular pyramid with base $B$ and vertex $V$* is the collection of all segments $VP$ for any point $P$ in $B$.

**GENERAL CONE:** Let $B$ be a region in a plane $E$ and $V$ be a point not in $E$. The *cone with base $B$ and vertex $V$* is the union of all segments $VP$ for all points $P$ in $B$ (See Figures 1 and 2).
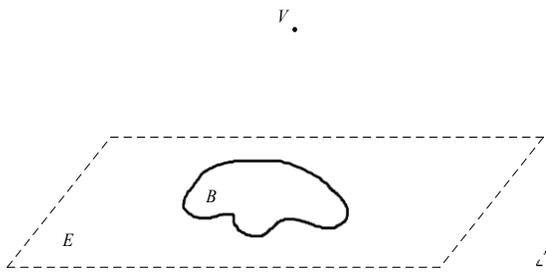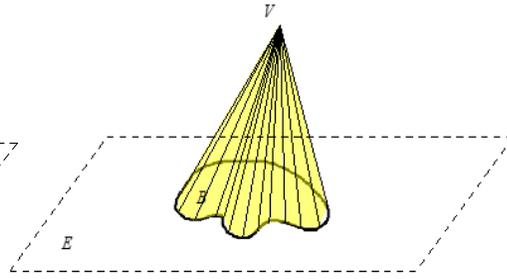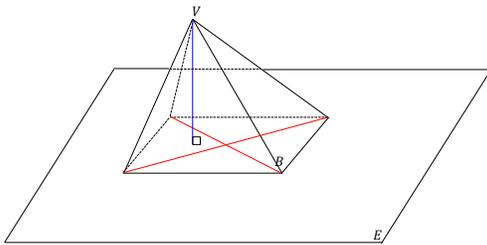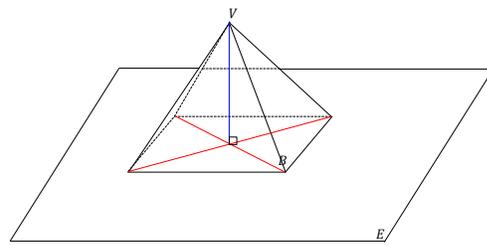
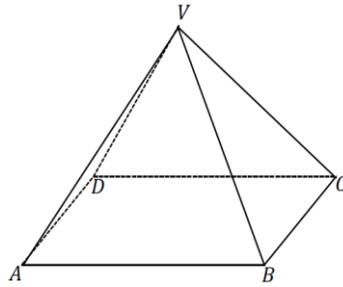Figure 1



Figure 2



Figure 3



Figure 4



Figure 5



Figure 6

### Example 1

In the following triangular pyramid, a plane passes through the pyramid so that it is parallel to the base and results in the cross-section $\triangle\, A'B'C'$.  If the area of $\triangle\, ABC$ is 25 mm$^2$, what is the area of $\triangle\, A'B'C'$?
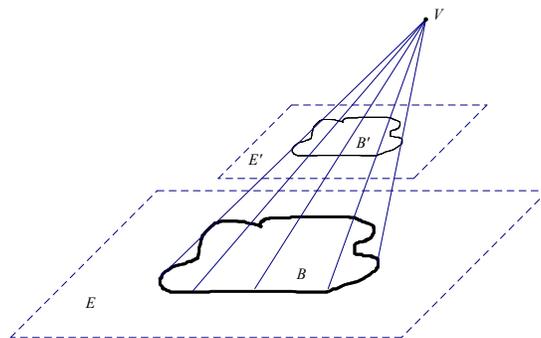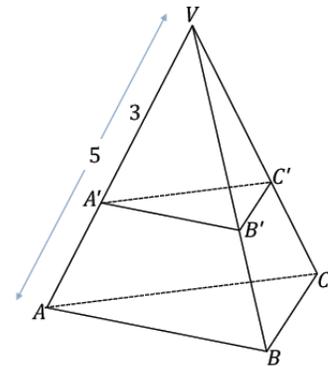


### Example 2

In the following triangular pyramid, a plane passes through the pyramid so that it is parallel to the base and results in the cross-section $\triangle\, A'B'C'$.  The altitude from $V$ is drawn; the intersection of the altitude with the base is $X$, and the intersection of the altitude with the cross-section is $X'$.  If the distance from $X$ to $V$ is 18 mm, the distance from $X'$ to $V$ is 12 mm, and the area of $\triangle\, A'B'C'$ is 28 mm$^2$, what is the area of $\triangle\, ABC$?
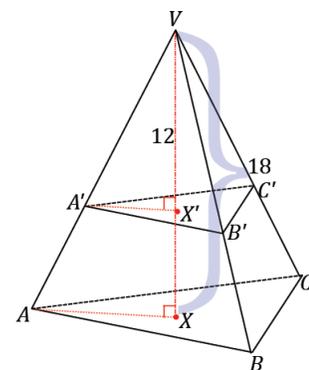
**Extension**
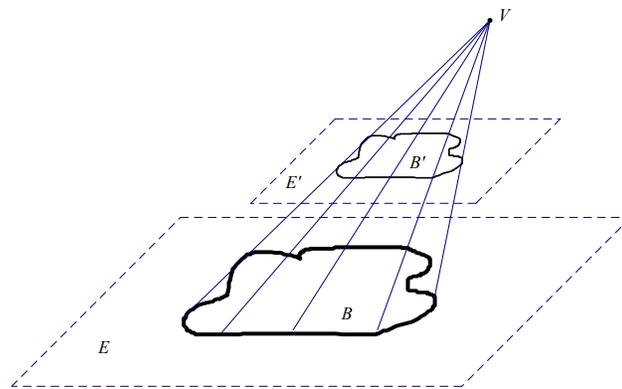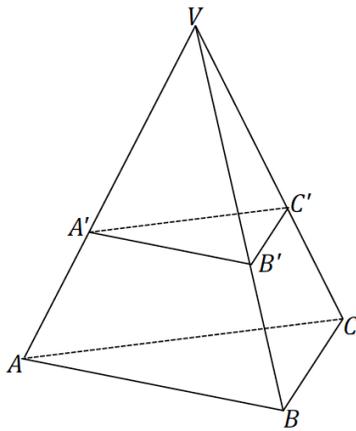


**Exercise 1**

The area of the base of a cone is 16, and the height is 10. Find the area of a cross-section that is distance 5 from the vertex.

## Example 3

**GENERAL CONE CROSS-SECTION THEOREM:**  If two general cones have the same base area and the same height, then cross-sections for the general cones the same distance from the vertex have the same area.

State the theorem in your own words.



**Figure 8**

Use the space below to prove the *general cone cross-section theorem*.
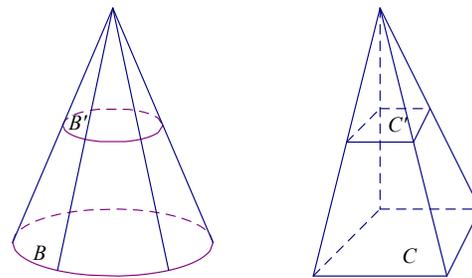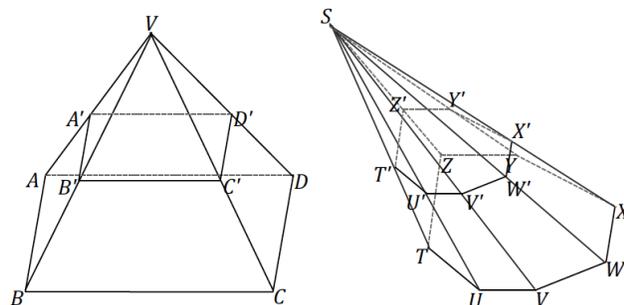
## Exercise 2

The following pyramids have equal altitudes, and both bases are equal in area and are coplanar.  Both pyramids' cross-sections are also coplanar.  If $BC = 3\sqrt{2}$ and $B'C' = 2\sqrt{3}$, and the area of $TUVWXYZ$ is 30 units$^2$, what is the area of cross-section $A'B'C'D'$?

**Lesson Summary**

**CONE:** Let $B$ be a region in a plane $E$ and $V$ be a point not in $E$. The *cone with base B and vertex V* is the union of all segments $VP$ for all points $P$ in $B$.

If the base is a polygonal region, then the *cone* is usually called a *pyramid*.

**RECTANGULAR PYRAMID:** Given a rectangular region $B$ in a plane $E$ and a point $V$ not in $E$, the *rectangular pyramid with base B and vertex V* is the union of all segments $VP$ for points $P$ in $B$.

**LATERAL EDGE AND FACE OF A PYRAMID:** Suppose the base $B$ of a pyramid with vertex $V$ is a polygonal region, and $P_i$ is a vertex of $B$. $\overline{P_iV}$ is called a *lateral edge* of the pyramid. If $\overline{P_iP_{i+1}}$ is a base edge of the base $B$ (a side of $B$), and $F$ is the union of all segments $PV$ for $P$ in $\overline{P_iP_{i+1}}$, then $F$ is called a *lateral face* of the pyramid. It can be shown that the face of a pyramid is always a triangular region.

**Problem Set**

1. The base of a pyramid has area 4. A cross-section that lies in a parallel plane that is distance of 2 from the base plane has an area of 1. Find the height, $h$, of the pyramid.
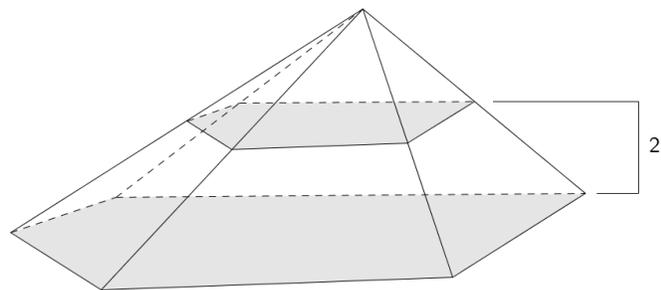


2. The base of a pyramid is a trapezoid. The trapezoidal bases have lengths of 3 and 5, and the trapezoid's height is 4. Find the area of the parallel slice that is three-fourths of the way from the vertex to the base.
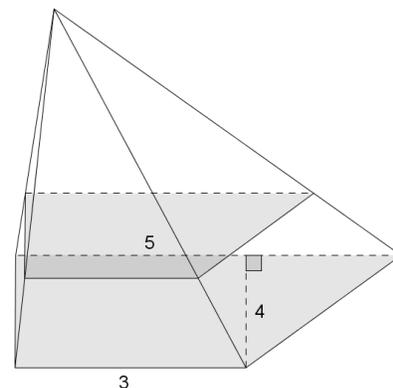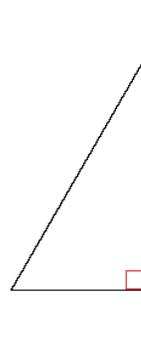
3. A cone has base area 36 cm². A parallel slice 5 cm from the vertex has area 25 cm². Find the height of the cone.

4. Sketch the figures formed if the triangular regions are rotated around the provided axis:
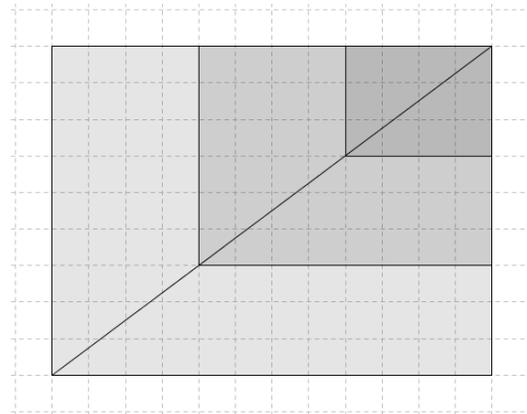
   a.
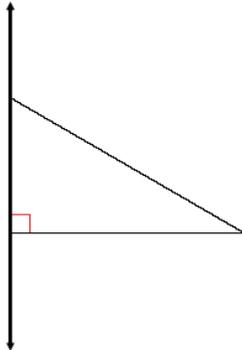
   b.

5. Liza drew the top view of a rectangular pyramid with two cross-sections as shown in the diagram and said that her diagram represents one, and only one, rectangular pyramid. Do you agree or disagree with Liza? Explain.

6. A general hexagonal pyramid has height 10 in. A slice 2 in. above the base has area 16 in². Find the area of the base.

7. A general cone has base area 3 units². Find the area of the slice of the cone that is parallel to the base and $\frac{2}{3}$ of the way from the vertex to the base.

8. A rectangular cone and a triangular cone have bases with the same area. Explain why the cross-sections for the cones halfway between the base and the vertex have the same area.
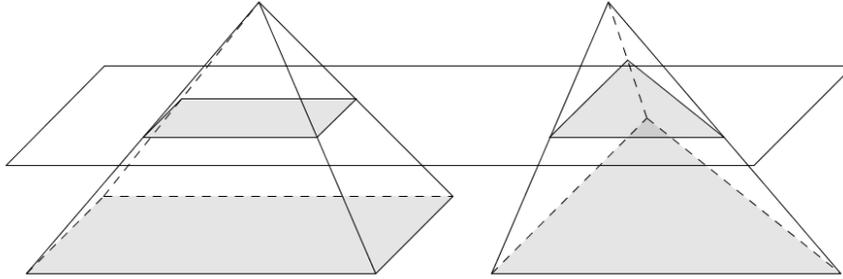


9. The following right triangle is rotated about side $AB$. What is the resulting figure, and what are its dimensions?