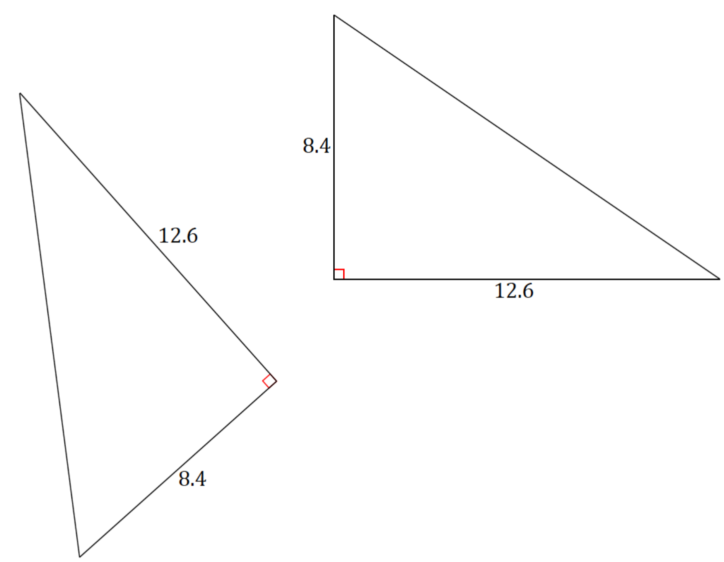
Lesson 2: Properties of Area

Classwork

Exploratory Challenge/Exercises 1–4

1. Two congruent triangles are shown below.



a. Calculate the area of each triangle.

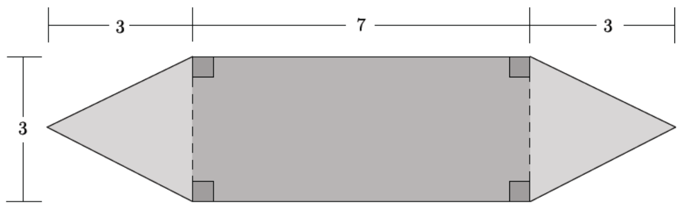
b. Circle the transformations that, if applied to the first triangle, would always result in a new triangle with the same area:

Translation Rotation Dilation Reflection

c. Explain your answer to part (b).

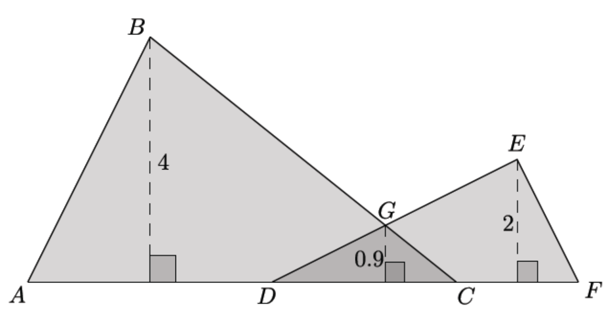
2.

a. Calculate the area of the shaded figure below.



b. Explain how you determined the area of the figure.

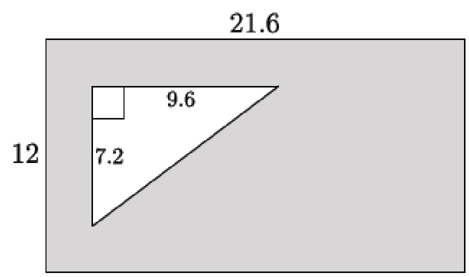
3. Two triangles $\triangle ABC$ and $\triangle DEF$ are shown below. The two triangles overlap forming $\triangle DGC$.



a. The base of figure $ABGEF$ is composed of segments of the following lengths: $AD = 4$, $DC = 3$, and $CF = 2$. Calculate the area of the figure $ABGEF$.

b. Explain how you determined the area of the figure.

4. A rectangle with dimensions 21.6×12 has a right triangle with a base 9.6 and a height of 7.2 cut out of the rectangle.



a. Find the area of the shaded region.

b. Explain how you determined the area of the shaded region.

Lesson Summary

SET (description): A *set* is a well-defined collection of objects called *elements* or *members* of the set.

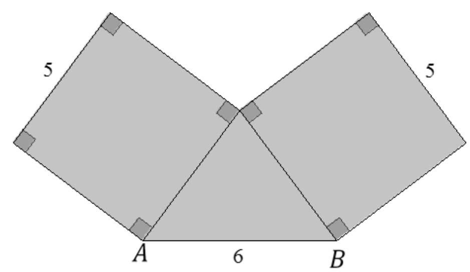
SUBSET: A set A is a *subset* of a set B if every element of A is also an element of B. The notation $A \subseteq B$ indicates that the set A is a subset of set B.

UNION: The *union* of A and B is the set of all objects that are either elements of A or of B, or of both. The union is denoted $A \cup B$.

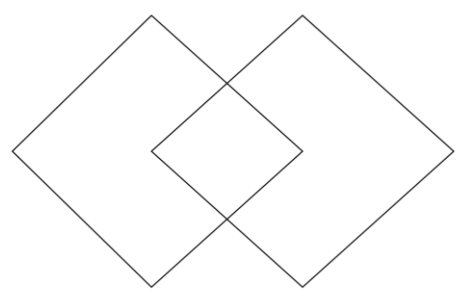
INTERSECTION: The *intersection* of A and B is the set of all objects that are elements of A and also elements of B. The intersection is denoted $A \cap B$.

Problem Set

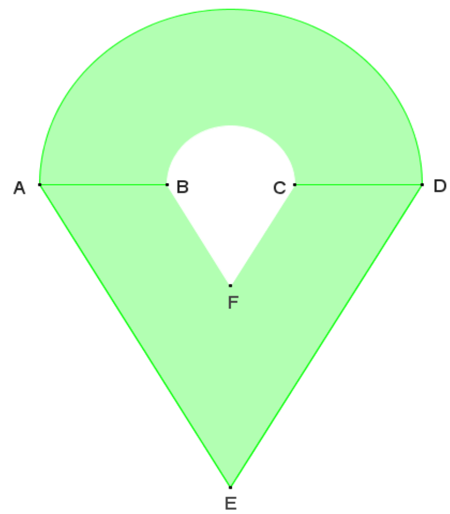
1. Two squares with side length 5 meet at a vertex and together with segment AB form a triangle with base 6 as shown. Find the area of the shaded region.



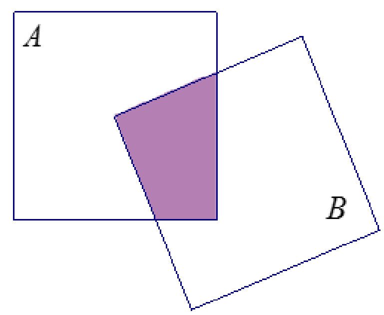
2. If two 2×2 square regions S_1 and S_2 meet at midpoints of sides as shown, find the area of the square region, $S_1 \cup S_2$.



3. The figure shown is composed of a semicircle and a non-overlapping equilateral triangle, and contains a hole that is also composed of a semicircle and a non-overlapping equilateral triangle. If the radius of the larger semicircle is 8, and the radius of the smaller semicircle is $\frac{1}{3}$ that of the larger semicircle, find the area of the figure.



4. Two square regions A and B each have $\text{Area}(8)$. One vertex of square B is the center point of square A. Can you find the area of $A \cup B$ and $A \cap B$ without any further information? What are the possible areas?



5. Four congruent right triangles with leg lengths a and b and hypotenuse length c are used to enclose the green region in Figure 1 with a square and then are rearranged inside the square leaving the green region in Figure 2.

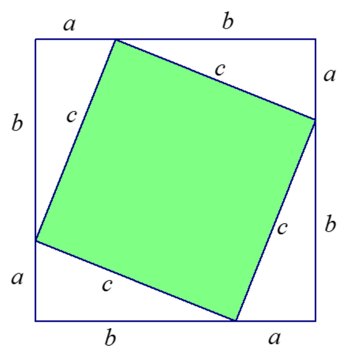


Figure 1

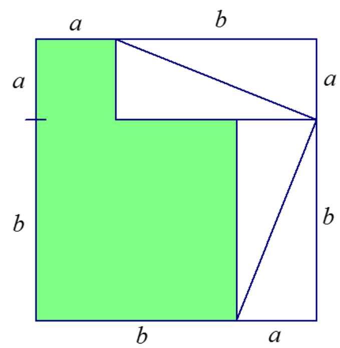


Figure 2

a. Use Property 4 to explain why the green region in Figure 1 has the same area as the green region in Figure 2.

b. Show that the green region in Figure 1 is a square, and compute its area.

c. Show that the green region in Figure 2 is the union of two non-overlapping squares, and compute its area.

d. How does this prove the Pythagorean theorem?