

Lesson 18: Similarity and the Angle Bisector Theorem

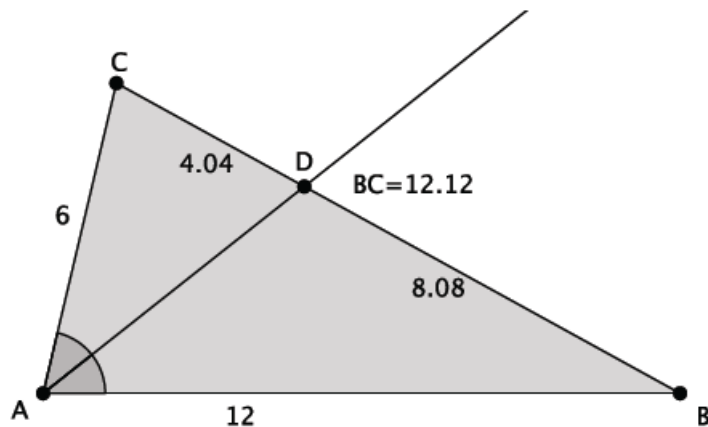
Classwork

Opening Exercise

- What is an angle bisector?
- Describe the angle relationships formed when parallel lines are cut by a transversal.
- What are the properties of an isosceles triangle?

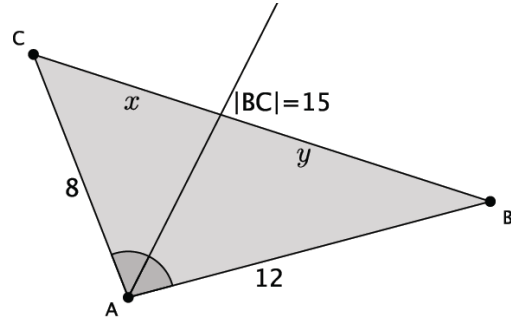
Discussion

In the diagram below, the angle bisector of $\angle A$ in $\triangle ABC$ meets side \overline{BC} at point D . Does the angle bisector create any observable relationships with respect to the side lengths of the triangle?

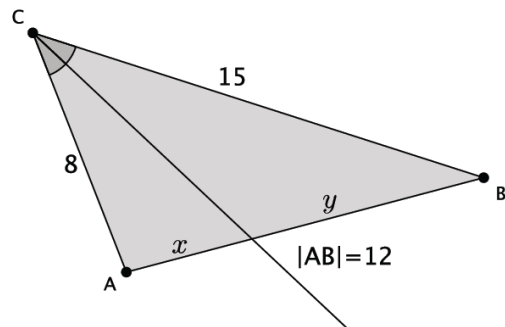


Exercises 1–4

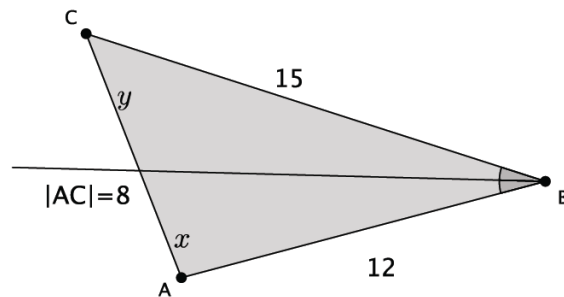
- The sides of a triangle are 8, 12, and 15. An angle bisector meets the side of length 15. Find the lengths x and y . Explain how you arrived at your answers.



- The sides of a triangle are 8, 12, and 15. An angle bisector meets the side of length 12. Find the lengths x and y .



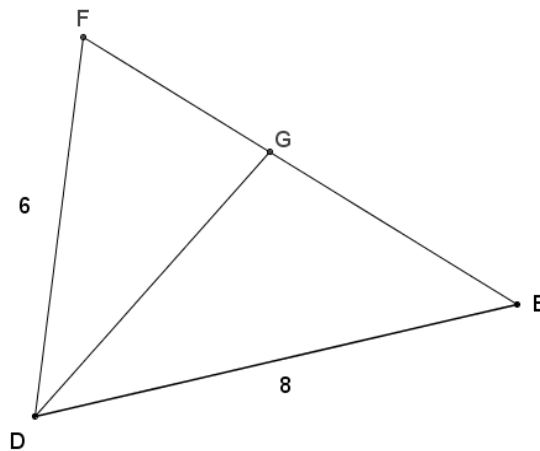
3. The sides of a triangle are 8, 12, and 15. An angle bisector meets the side of length 8. Find the lengths x and y .



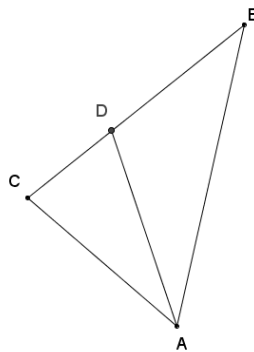
4. The angle bisector of an angle splits the opposite side of a triangle into lengths 5 and 6. The perimeter of the triangle is 33. Find the lengths of the other two sides.

Problem Set

- The sides of a triangle have lengths of 5, 8, and $6\frac{1}{2}$. An angle bisector meets the side of length $6\frac{1}{2}$. Find the lengths x and y .
- The sides of a triangle are $10\frac{1}{2}$, $16\frac{1}{2}$, and 9. An angle bisector meets the side of length 9. Find the lengths x and y .
- In the diagram of triangle DEF below, \overline{DG} is an angle bisector, $DE = 8$, $DF = 6$, and $EF = 8\frac{1}{6}$. Find FG and EG .

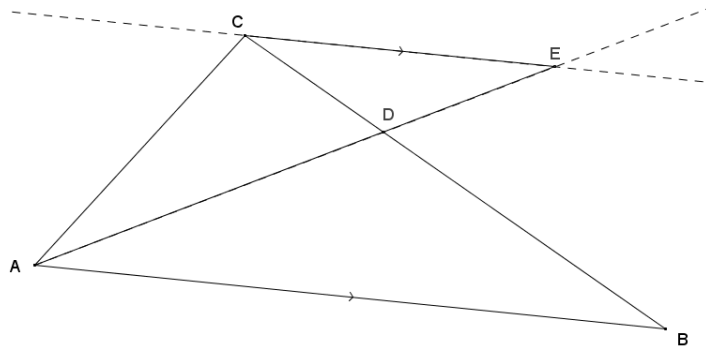


- Given the diagram below and $\angle BAD \cong \angle DAC$, show that $BD:BA = CD:CA$.



- The perimeter of triangle LMN is 32 cm. \overline{NX} is the angle bisector of angle N , $LX = 3$ cm, and $XM = 5$ cm. Find LN and MN .
- Given $CD = 3$, $DB = 4$, $BF = 4$, $FE = 5$, $AB = 6$, and $\angle CAD \cong \angle DAB \cong \angle BAF \cong \angle FAE$, find the perimeter of quadrilateral $AEBC$.

7. If \overline{AE} meets \overline{BC} at D such that $CD:BD = CA:BA$, show that $\angle CAD \cong \angle BAD$. Explain how this proof relates to the angle bisector theorem.



8. In the diagram below, $\overline{ED} \cong \overline{DB}$, \overline{BE} bisects $\angle ABC$, $AD = 4$, and $DC = 8$. Prove that $\triangle ADB \sim \triangle CEB$.

