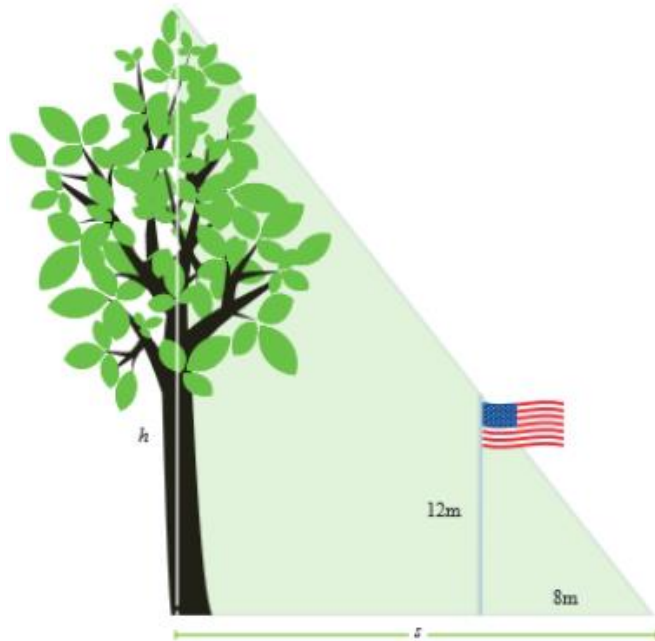


## Lesson 16: Between-Figure and Within-Figure Ratios

### Classwork

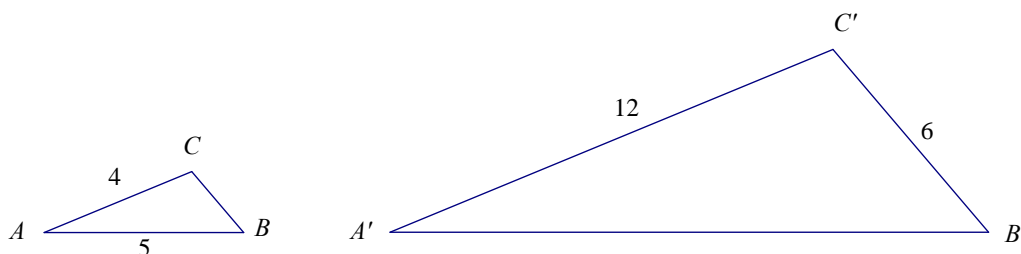
#### Opening Exercise

At a certain time of day, a 12 m flagpole casts an 8 m shadow. Write an equation that would allow you to find the height,  $h$ , of the tree that uses the length,  $s$ , of the tree's shadow.

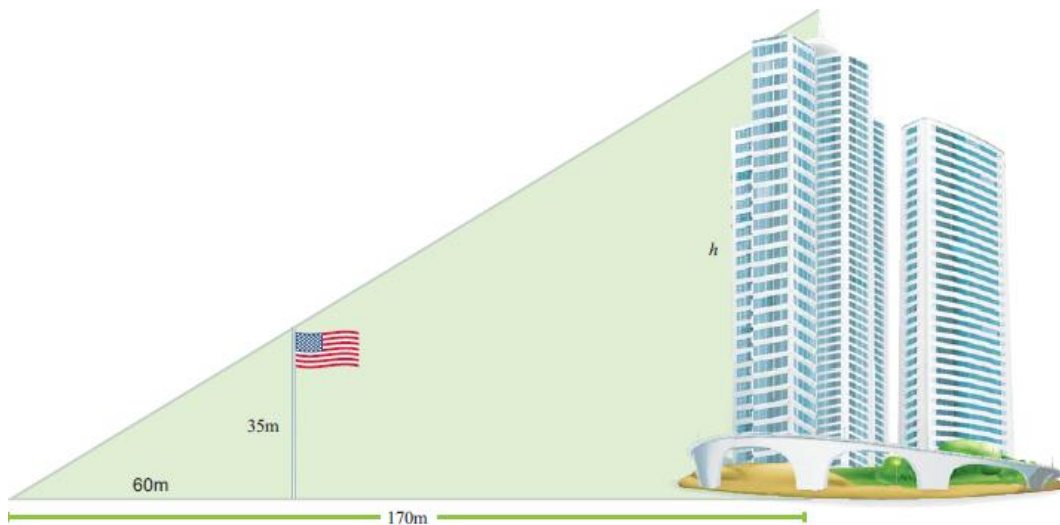


#### Example 1

Given  $\triangle ABC \sim \triangle A'B'C'$ , find the missing side lengths.



## Example 2

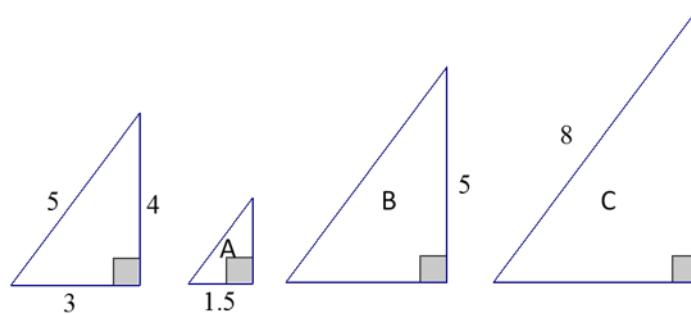


In the diagram above, a large flagpole stands outside of an office building. Marquis realizes that when he looks up from the ground 60 m away from the flagpole, the top of the flagpole and the top of the building line up. If the flagpole is 35 m tall and Marquis is 170 m from the building, how tall is the building?

- Are the triangles in the diagram similar? Explain.
- Determine the height of the building using what you know about scale factors.
- Determine the height of the building using ratios *between* similar figures.
- Determine the height of the building using ratios *within* similar figures.

**Example 3**

The following right triangles are similar. We will determine the unknown side lengths by using ratios within the first triangle. For each of the triangles below, we define the base as the horizontal length of the triangle and the height as the vertical length.



- Write and find the value of the ratio that compares the height to the hypotenuse of the leftmost triangle.
- Write and find the value of the ratio that compares the base to the hypotenuse of the leftmost triangle.
- Write and find the value of the ratio that compares the height to the base of the leftmost triangle.
- Use the triangle with lengths 3–4–5 and triangle *A* to answer the following questions:
  - Which ratio can be used to determine the height of triangle *A*?
  - Which ratio can be used to determine the hypotenuse of triangle *A*?
  - Find the unknown lengths of triangle *A*.

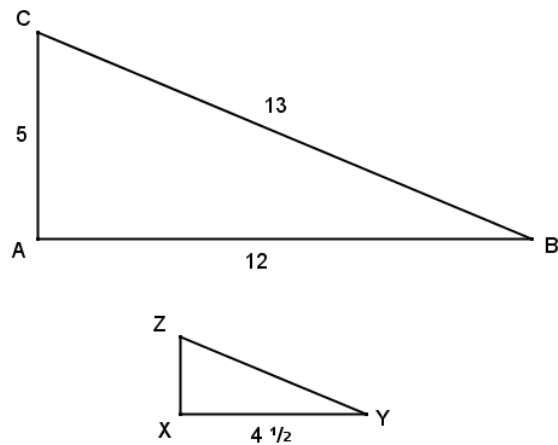
- e. Use the triangle with lengths 3–4–5 and triangle  $B$  to answer the following questions:
- Which ratio can be used to determine the base of triangle  $B$ ?
  - Which ratio can be used to determine the hypotenuse of triangle  $B$ ?
  - Find the unknown lengths of triangle  $B$ .
- f. Use the triangle with lengths 3–4–5 and triangle  $C$  to answer the following questions:
- Which ratio can be used to determine the height of triangle  $C$ ?
  - Which ratio can be used to determine the base of triangle  $C$ ?
  - Find the unknown lengths of triangle  $C$ .
- g. Explain the relationship of the ratio of the corresponding sides within a figure to the ratio of the corresponding sides within a similar figure.
- h. How does the relationship you noted in part (g) allow you to determine the length of an unknown side of a triangle?

**Problem Set**

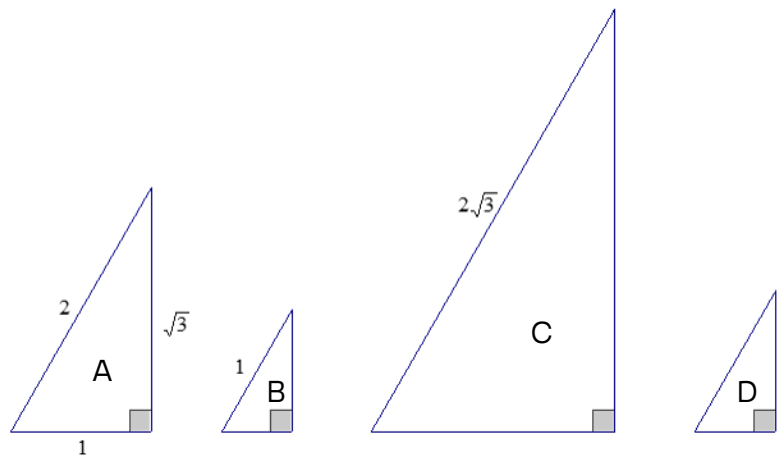
1.  $\triangle DEF \sim \triangle ABC$  All side length measurements are in centimeters. Use between-figure ratios and/or within-figure ratios to determine the unknown side lengths.

2. Given  $\triangle ABC \sim \triangle XYZ$ , answer the following questions:

- Write and find the value of the ratio that compares the height  $\overline{AC}$  to the hypotenuse of  $\triangle ABC$ .
- Write and find the value of the ratio that compares the base  $\overline{AB}$  to the hypotenuse of  $\triangle ABC$ .
- Write and find the value of the ratio that compares the height  $\overline{AC}$  to the base  $\overline{AB}$  of  $\triangle ABC$ .
- Use within-figure ratios to find the corresponding height of  $\triangle XYZ$ .
- Use within-figure ratios to find the hypotenuse of  $\triangle XYZ$ .

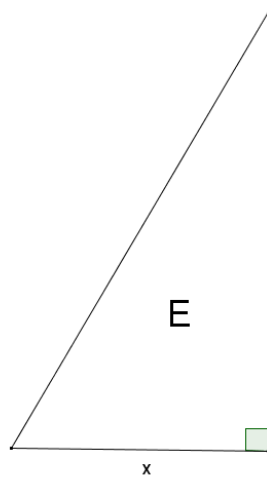


3. Right triangles  $A$ ,  $B$ ,  $C$ , and  $D$  are similar. Determine the unknown side lengths of each triangle by using ratios of side lengths within triangle  $A$ .

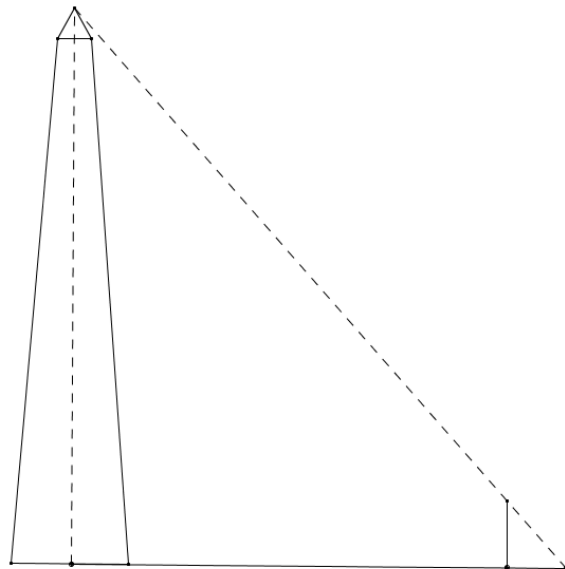


- Write and find the value of the ratio that compares the height to the hypotenuse of triangle  $A$ .
- Write and find the value of the ratio that compares the base to the hypotenuse of triangle  $A$ .
- Write and find the value of the ratio that compares the height to the base of triangle  $A$ .
- Which ratio can be used to determine the height of triangle  $B$ ? Find the height of triangle  $B$ .
- Which ratio can be used to determine the base of triangle  $B$ ? Find the base of triangle  $B$ .
- Find the unknown lengths of triangle  $C$ .
- Find the unknown lengths of triangle  $D$ .

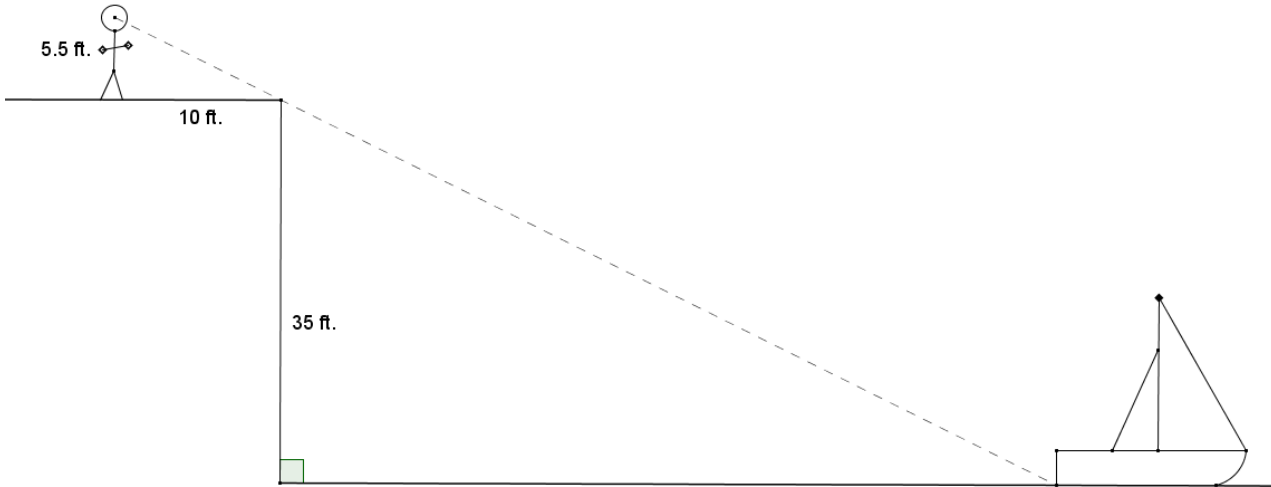
- h. Triangle  $E$  is also similar to triangles  $A, B, C,$  and  $D$ . Find the lengths of the missing sides in terms of  $x$ .



4. Brian is photographing the Washington Monument and wonders how tall the monument is. Brian places his 5 ft. camera tripod approximately 100 yd. from the base of the monument. Lying on the ground, he visually aligns the top of his tripod with the top of the monument and marks his location on the ground approximately 2 ft. 9 in. from the center of his tripod. Use Brian’s measurements to approximate the height of the Washington Monument.



5. Catarina’s boat has come untied and floated away on the lake. She is standing atop a cliff that is 35 ft. above the water in a lake. If she stands 10 ft. from the edge of the cliff, she can visually align the top of the cliff with the water at the back of her boat. Her eye level is  $5\frac{1}{2}$  ft. above the ground. Approximately how far out from the cliff is Catarina’s boat?



6. Given the diagram below and  $\triangle ABC \sim \triangle XYZ$ , find the unknown lengths  $x$ ,  $2x$ , and  $3x$ .

