Lesson 14: Similarity

Classwork

Example 1

We said that for a figure A in the plane, it must be true that $A \sim A$. Describe why this must be true.

Example 2

We said that for figures A and B in the plane so that $A \sim B$, then it must be true that $B \sim A$. Describe why this must be true.

Example 3

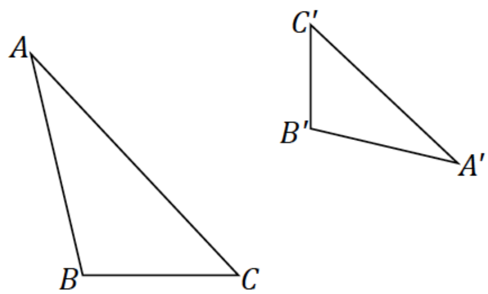
Based on the definition of *similar,* how would you show that any two circles are similar?

Example 4

Suppose $\triangle ABC \leftrightarrow \triangle DEF$ and that, under this correspondence, corresponding angles are equal and corresponding sides are proportional. Does this guarantee that $\triangle ABC$ and $\triangle DEF$ are similar?

Example 5

a. In the diagram below, $\triangle ABC \sim \triangle A'B'C'$. Describe a similarity transformation that maps $\triangle ABC$ to $\triangle A'B'C'$.



b. Joel says the sequence must require a dilation and three rigid motions, but Sharon is sure there is a similarity transformation composed of just a dilation and two rigid motions. Who is right?

Lesson Summary

Similarity is reflexive because a figure is similar to itself.

Similarity is symmetric because once a similarity transformation is determined to take a figure to another, there are inverse transformations that can take the figure back to the original.

Problem Set

1. If you are given any two congruent triangles, describe a sequence of basic rigid motions that takes one to the other.

2. If you are given two similar triangles that are not congruent triangles, describe a sequence of dilations and basic rigid motions that takes one to the other.

3. Given two line segments, $\overline{AB}$ and $\overline{CD}$, of different lengths, answer the following questions:
   a. It is always possible to find a similarity transformation that maps $\overline{AB}$ to $\overline{CD}$ sending A to C and B to D. Describe one such similarity transformation.
   b. If you are given that $\overline{AB}$ and $\overline{CD}$ are not parallel, are not congruent, do not share any points, and do not lie in the same line, what is the fewest number of transformations needed in a sequence to map $\overline{AB}$ to $\overline{CD}$? Which transformations make this work?
   c. If you performed a similarity transformation that instead takes A to D and B to C, either describe what mistake was made in the similarity transformation, or describe what additional transformation is needed to fix the error so that A maps to C and B maps to D.

4. We claim that similarity is transitive (i.e., if A, B, and C are figures in the plane such that $A \sim B$ and $B \sim C$, then $A \sim C$). Describe why this must be true.

5. Given two line segments, $\overline{AB}$ and $\overline{CD}$, of different lengths, we have seen that it is always possible to find a similarity transformation that maps $\overline{AB}$ to $\overline{CD}$, sending A to C and B to D with one rotation and one dilation. Can you do this with one reflection and one dilation?

6. Given two triangles, $\triangle ABC \sim \triangle DEF$, is it always possible to rotate $\triangle ABC$ so that the sides of $\triangle ABC$ are parallel to the corresponding sides in $\triangle DEF$ (e.g., $\overline{AB} \parallel \overline{DE}$)?