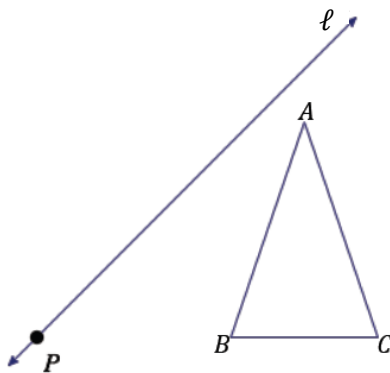# Lesson 13: Properties of Similarity Transformations

**Classwork**

Example 1

Similarity transformation G consists of a rotation about the point P by $90°$, followed by a dilation centered at P with a scale factor of $r = 2$, and then followed by a reflection across line ℓ. Find the image of the triangle.

Example 2

A similarity transformation G applied to trapezoid $ABCD$ consists of a translation by vector $\overrightarrow{XY}$, followed by a reflection across line m, and then followed by a dilation centered at P with a scale factor of $r = 2$. Recall that we can describe the same sequence using the following notation: $D_{P,2}\left(r_m\left(T_{XY}(ABCD)\right)\right)$. Find the image of $ABCD$.
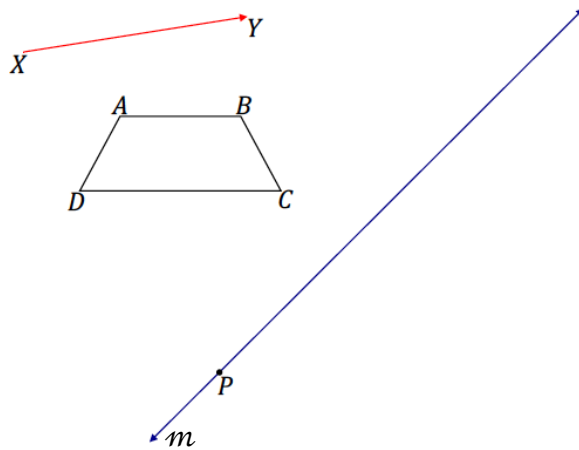
Exercise 1

A similarity transformation for triangle DEF is described by $r_n\left(D_{A,\frac{1}{2}}\left(R_{A,90°}(DEF)\right)\right)$. Locate and label the image of triangle DEF under the similarity.
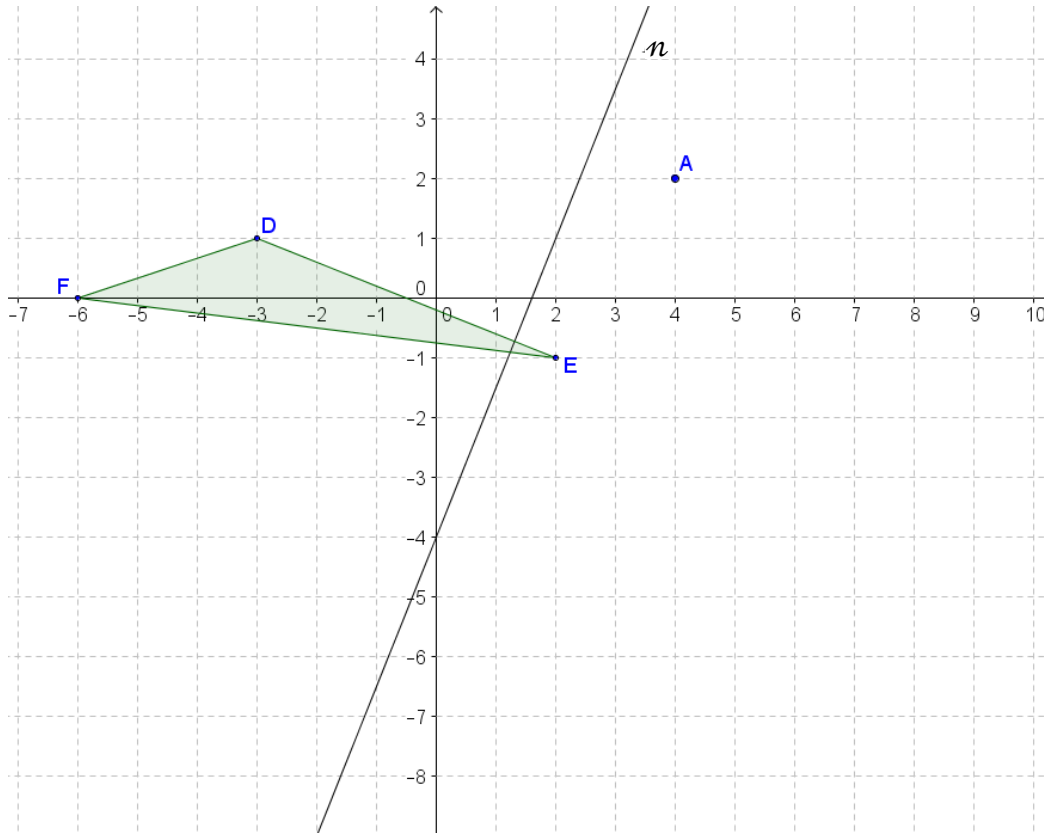
Lesson Summary

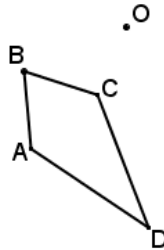Properties of similarity transformations:

1. Distinct points are mapped to distinct points.

2. Each point P' in the plane has a pre-image.

3. There is a scale factor of r for G so that for any pair of points P and Q with images $P' = G(P)$ and $Q' = G(Q)$, then $P'Q' = rPQ$.

4. A similarity transformation sends lines to lines, rays to rays, line segments to line segments, and parallel lines to parallel lines.

5. A similarity transformation sends angles to angles of equal measure.

6. A similarity transformation maps a circle of radius R to a circle of radius rR, where r is the scale factor of the similarity transformation.
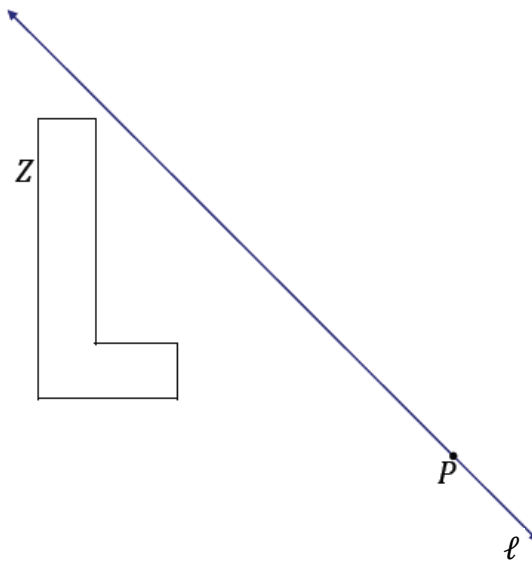

Problem Set

1. A similarity transformation consists of a reflection over line ℓ, followed by a dilation from O with a scale factor of $r = \dfrac{3}{4}$. Use construction tools to find $\triangle\, G''H''I''$.
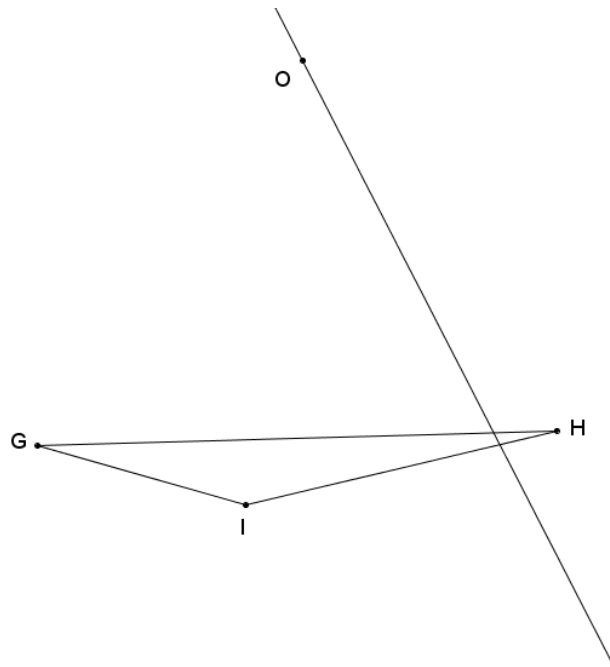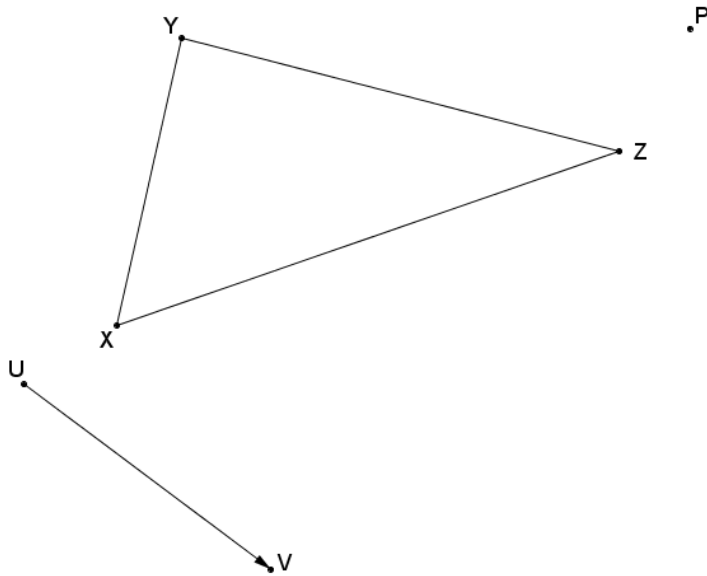
2. A similarity transformation consists of a dilation from point O with a scale factor of $r = 2\frac{1}{2}$, followed by a rotation about O of $-90°$. Use construction tools to find kite $A''B''C''D''$.

3. For the Figure Z, find the image of $r_\ell\left(R_{P,90°}\left(D_{P,\frac{1}{2}}(Z)\right)\right)$.

4. A similarity transformation consists of a translation along vector $\overrightarrow{UV}$, followed by a rotation of $60°$ about P, then followed by a dilation from P with a scale factor of $r = \frac{1}{3}$. Use construction tools to find $\triangle\, X'''Y'''Z'''$.

5. Given the quarter-circular figure determined by points A, B, and C, a similarity transformation consists of a $-65°$ rotation about point B, followed by a dilation from point O with a scale factor of $r = \frac{1}{2}$. Find the image of the figure determined by points A'', B'', and C''.
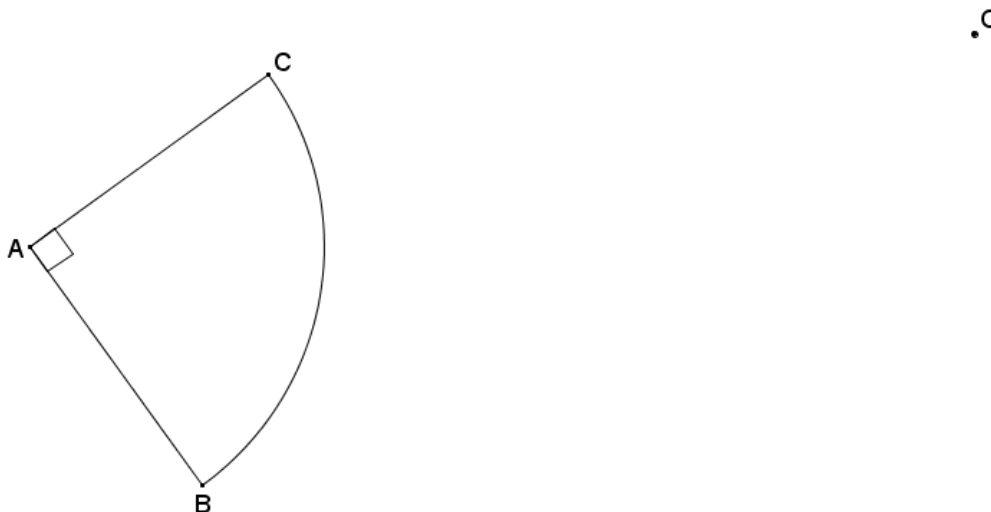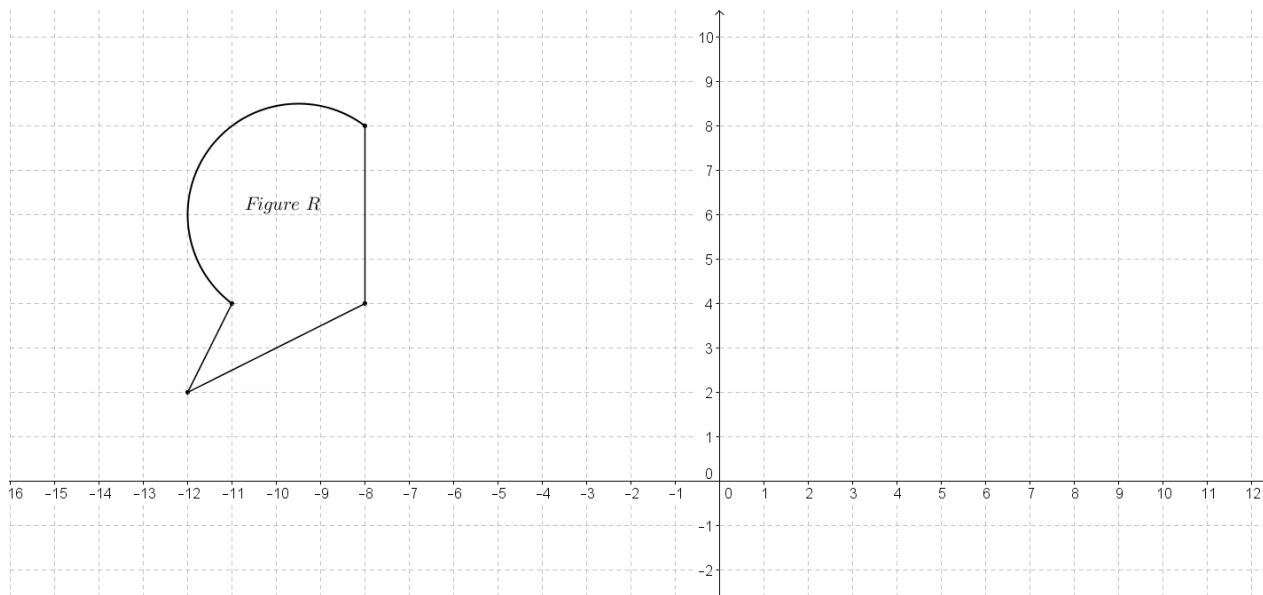
Describe a different similarity transformation that would map quarter-circle ABC to quarter-circle $A''B''C''$.

6. A similarity transformation consists of a dilation from center O with a scale factor of $\frac{1}{2}$, followed by a rotation of 60° about point O. Complete the similarity transformation on Figure T to complete the drawing of Figure T''.
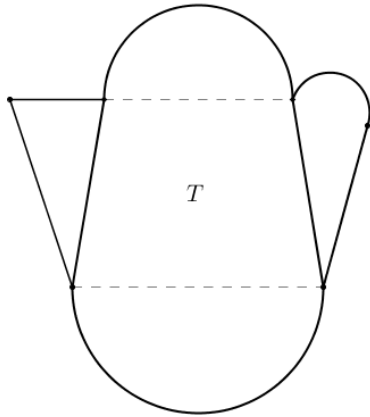
7. Given Figure R on the coordinate plane shown below, a similarity transformation consists of a dilation from $(0,6)$ with a scale factor of $\frac{1}{4}$, followed by a reflection over line $x = -1$, and then followed by a vertical translation of 5 units down. Find the image of Figure R.

8. Given $\triangle ABC$, with vertices $A(2, -7)$, $B(-2, -1)$, and $C(3, -4)$, locate and label the image of the triangle under the similarity transformation $D_{B', \frac{1}{2}}\left(R_{A, 120°}\left(r_{x=2}(ABC)\right)\right)$.

9. In Problem 8, describe the relationship of A''' to $\overline{AB'}$, and explain your reasoning.

10. Given $O(-8,3)$ and quadrilateral $BCDE$, with $B(-5,1)$, $C(-6,-1)$, $D(-4,-1)$, and $E(-4,2)$, what are the coordinates of the vertices of the image of $BCDE$ under the similarity transformation $r_{x-axis}\left(D_{O,3}(BCDE)\right)$?

11. Given triangle ABC as shown on the diagram of the coordinate plane:

    a. Perform a translation so that vertex A maps to the origin.

    b. Next, dilate the image $A'B'C'$ from the origin using a scale factor of $\frac{1}{3}$.

    c. Finally, translate the image $A''B''C''$ so that the vertex A'' maps to the original point A.

    d. Using transformations, describe how the resulting image $A'''B''C''$ relates to the original figure ABC.
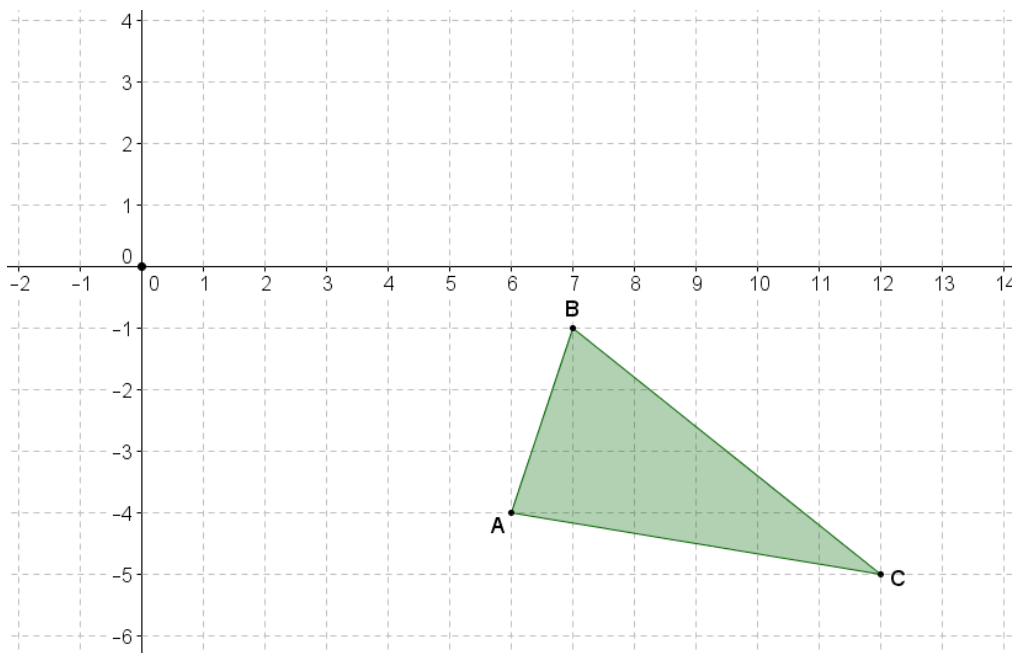
12.

   a. In the coordinate plane, name the single transformation resulting from the composition of the two dilations shown below:

   $D_{(0,0),2}$ followed by $D_{(0,4),\frac{1}{2}}$

   (Hint: Try it!)

   b. In the coordinate plane, name the single transformation resulting from the composition of the two dilations shown below:

   $D_{(0,0),2}$ followed by $D_{(4,4),\frac{1}{2}}$

   (Hint: Try it!)

   c. Using the results from parts (a) and (b), describe what happens to the origin under both similarity transformations.