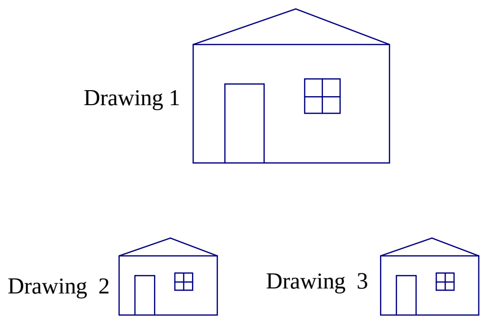
Lesson 11: Dilations from Different Centers

Classwork

Exploratory Challenge 1

Drawing 2 and Drawing 3 are both scale drawings of Drawing 1.



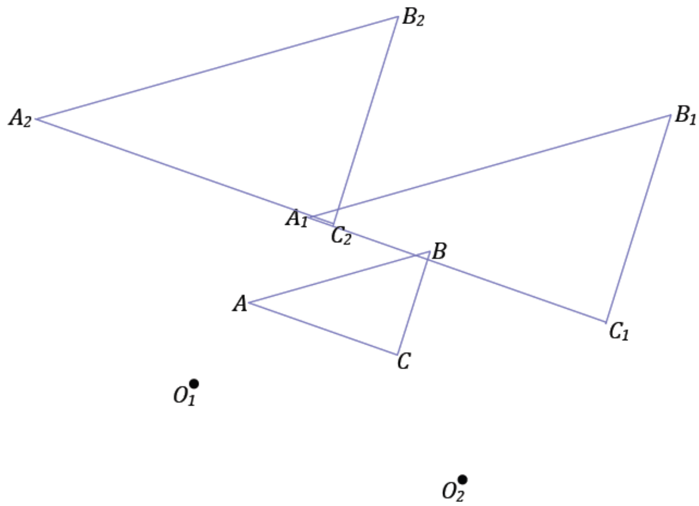
a. Determine the scale factor and center for each scale drawing. Take measurements as needed.

b. Is there a way to map Drawing 2 onto Drawing 3 or map Drawing 3 onto Drawing 2?

c. Generalize the parameters of this example and its results.

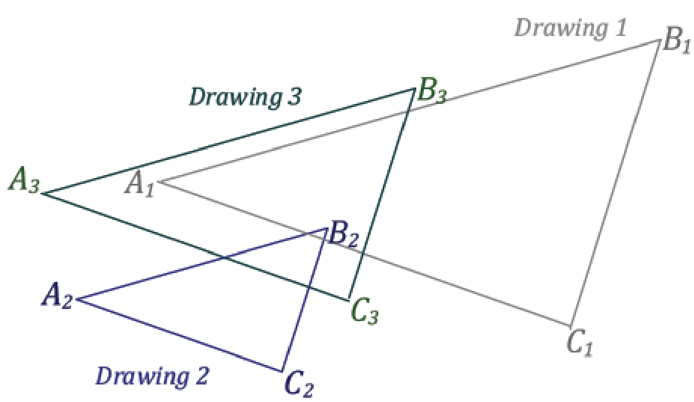
Exercise 1

Triangle ABC has been dilated with scale factor $\frac{1}{2}$ from centers O_1 and O_2. What can you say about line segments A_1A_2, B_1B_2, and C_1C_2?



Exploratory Challenge 2

If Drawing 2 is a scale drawing of Drawing 1 with scale factor r_1 and Drawing 3 is a scale drawing of Drawing 2 with scale factor r_2, what is the relationship between Drawing 3 and Drawing 1?



a. Determine the scale factor and center for each scale drawing. Take measurements as needed.

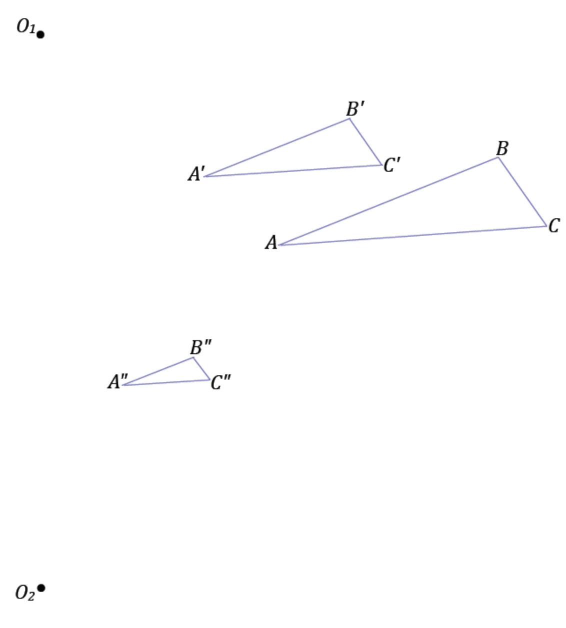
b. What is the scale factor going from Drawing 1 to Drawing 3? Take measurements as needed.

c. Compare the centers of dilations of Drawing 1 (to Drawing 2) and of Drawing 2 (to Drawing 3). What do you notice about these centers relative to the center of the composition of dilations O_3?

d. Generalize the parameters of this example and its results.

Exercise 2

Triangle ABC has been dilated with scale factor $\frac{2}{3}$ from center O_1 to get triangle $A'B'C'$, and then triangle $A'B'C'$ is dilated from center O_2 with scale factor $\frac{1}{2}$ to get triangle $A''B''C''$. Describe the dilation that maps triangle ABC to triangle $A''B''C''$. Find the center and scale factor for that dilation.



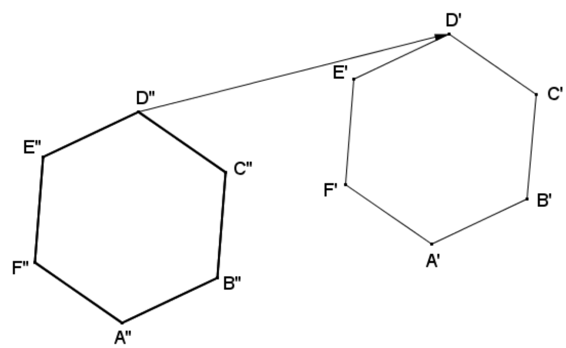
Lesson Summary

In a series of dilations, the scale factor that maps the original figure onto the final image is the product of all the scale factors in the series of dilations.

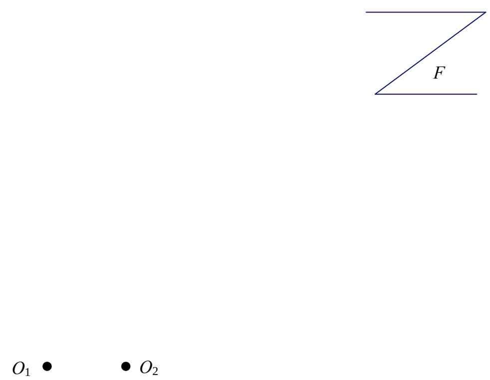
Problem Set

1. In Lesson 7, the dilation theorem for line segments said that if two different-length line segments in the plane were parallel to each other, then a dilation exists mapping one segment onto the other. Explain why the line segments must be different lengths for a dilation to exist.

2. Regular hexagon $A'B'C'D'E'F'$ is the image of regular hexagon $ABCDEF$ under a dilation from center O_1, and regular hexagon $A''B''C''D''E''F''$ is the image of regular hexagon $ABCDEF$ under a dilation from center O_2. Points A', B', C', D', E', and F' are also the images of points A'', B'', C'', D'', E'', and F'', respectively, under a translation along vector $\overrightarrow{D''D'}$. Find a possible regular hexagon $ABCDEF$.

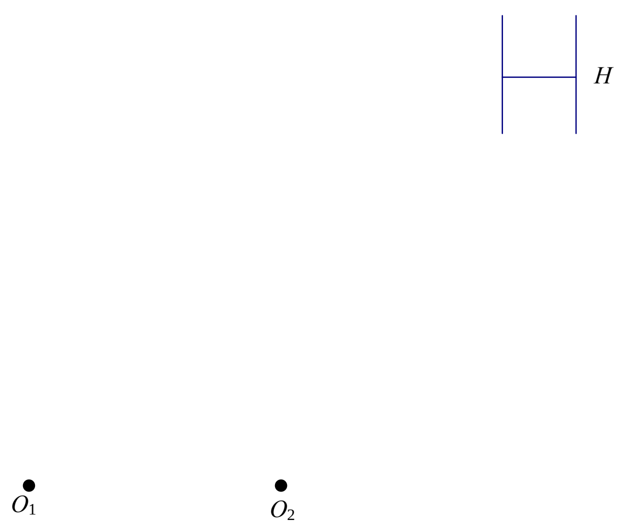


3. A dilation with center O_1 and scale factor $\frac{1}{2}$ maps figure F to figure F'. A dilation with center O_2 and scale factor $\frac{3}{2}$ maps figure F' to figure F''. Draw figures F' and F'', and then find the center O and scale factor r of the dilation that takes F to F''.

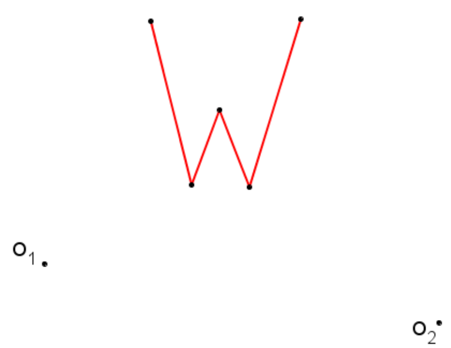


4. A figure T is dilated from center O_1 with a scale factor $r_1 = \frac{3}{4}$ to yield image T', and figure T' is then dilated from center O_2 with a scale factor $r_2 = \frac{4}{3}$ to yield figure T''. Explain why $T \cong T''$.

5. A dilation with center O_1 and scale factor $\frac{1}{2}$ maps figure H to figure H'. A dilation with center O_2 and scale factor 2 maps figure H' to figure H''. Draw figures H' and H''. Find a vector for a translation that maps H to H''.



6. Figure W is dilated from O_1 with a scale factor $r_1 = 2$ to yield W'. Figure W' is then dilated from center O_2 with a scale factor $r_2 = \frac{1}{4}$ to yield W''.

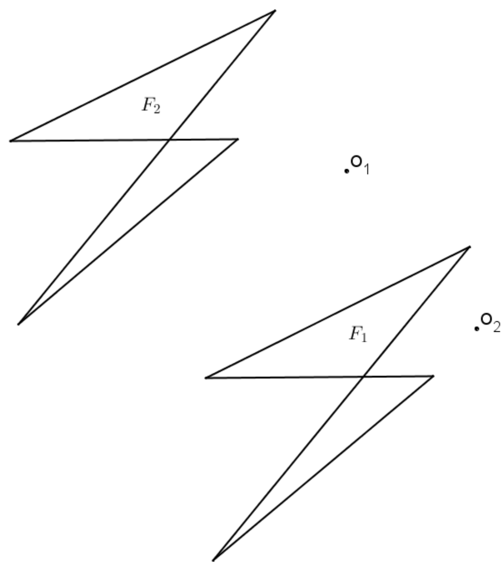


a. Construct the composition of dilations of figure W described above.

b. If you were to dilate figure W'', what scale factor would be required to yield an image that is congruent to figure W?

c. Locate the center of dilation that maps W'' to W using the scale factor that you identified in part (b).

7. Figures F_1 and F_2 in the diagram below are dilations of F from centers O_1 and O_2, respectively.



   a. Find F.
   b. If $F_1 \cong F_2$, what must be true of the scale factors r_1 and r_2 of each dilation?
   c. Use direct measurement to determine each scale factor for D_{O_1,r_1} and D_{O_2,r_2}.

8. Use a coordinate plane to complete each part below using $U(2,3)$, $V(6,6)$, and $W(6,-1)$.
   a. Dilate $\triangle UVW$ from the origin with a scale factor $r_1 = 2$. List the coordinates of image points U', V', and W'.
   b. Dilate $\triangle UVW$ from $(0,6)$ with a scale factor of $r_2 = \frac{3}{4}$. List the coordinates of image points U'', V'', and W''.
   c. Find the scale factor, r_3, from $\triangle U'V'W'$ to $\triangle U''V''W''$.
   d. Find the coordinates of the center of dilation that maps $\triangle U'V'W'$ to $\triangle U''V''W''$.