Lesson 5: Scale Factors

Classwork

Opening Exercise

Quick Write: Describe how a figure is transformed under a dilation with a scale factor $= 1$, $r > 1$, and $0 < r < 1$.

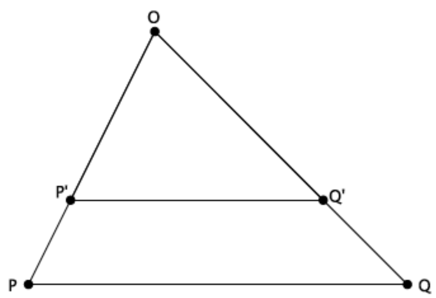
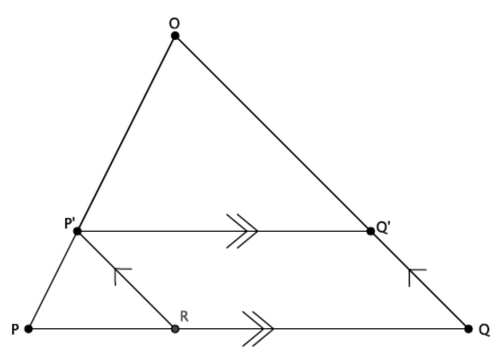
Discussion

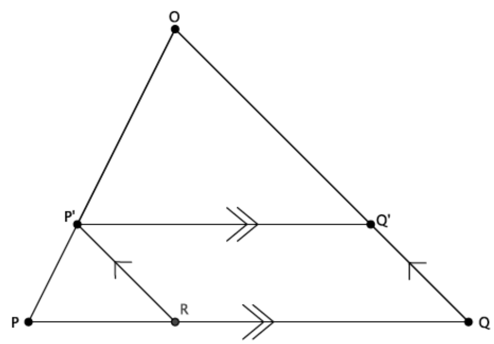
DILATION THEOREM: If a dilation with center O and scale factor r sends point P to P' and Q to Q', then $|P'Q'| = r|PQ|$. Furthermore, if $r \neq 1$ and O, P, and Q are the vertices of a triangle, then $\overleftrightarrow{PQ} || \overleftrightarrow{P'Q'}$.

Now consider the dilation theorem when O, P, and Q are the vertices of $\triangle OPQ$. Since P' and Q' come from a dilation with scale factor r and center O, we have $\frac{OP'}{OP} = \frac{OQ'}{OQ} = r$.

There are two cases that arise; recall what you wrote in your Quick Write. We must consider the case when $r > 1$ and when $0 < r < 1$. Let's begin with the latter.

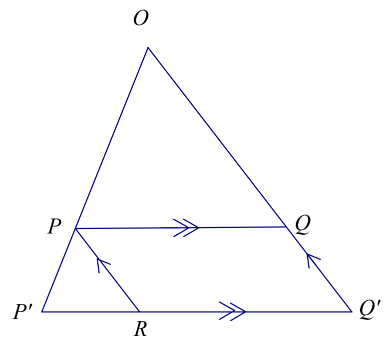
Dilation Theorem Proof, Case 1

Statements	Reasons/Explanations
	
1. A dilation with center O and scale factor r sends point P to P' and Q to Q'.	1.
2. $\frac{OP\prime}{OP} = \frac{OQ\prime}{OQ} = r$	2.
3. $\overleftrightarrow{PQ} \| \overleftrightarrow{P'Q'}$	3.
4. A dilation with center P and scale factor $\frac{PP'}{PO}$ sends point O to P' and point Q to R. Draw $\overline{P'R}$.	4.
	
5. $\overline{P'R} \| \overline{OQ'}$	5.

6. $RP'Q'Q$ is a parallelogram. 	6.
7. $RQ = P'Q'$	7.
8. $\frac{RQ}{PQ} = \frac{P'O}{PO}$	8.
9. $\frac{RQ}{PQ} = r$	9.
10. $RQ = r \cdot PQ$	10.
11. $P'Q' = r \cdot PQ$	11.

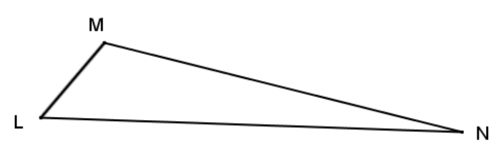
Exercises

1. Prove Case 2: If O, P, and Q are the vertices of a triangle and $r > 1$, show that (a) $\overleftrightarrow{PQ} \parallel \overleftrightarrow{P'Q'}$ and (b) $P'Q' = rPQ$. Use the diagram below when writing your proof.



2.

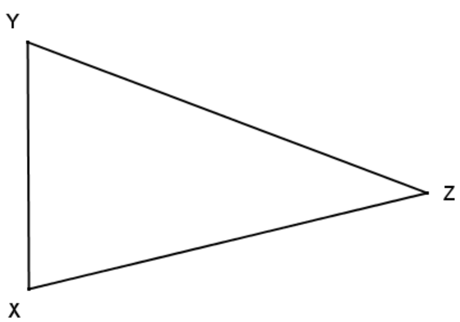
   a. Produce a scale drawing of $\triangle LMN$ using either the ratio or parallel method with point M as the center and a scale factor of $\frac{3}{2}$.



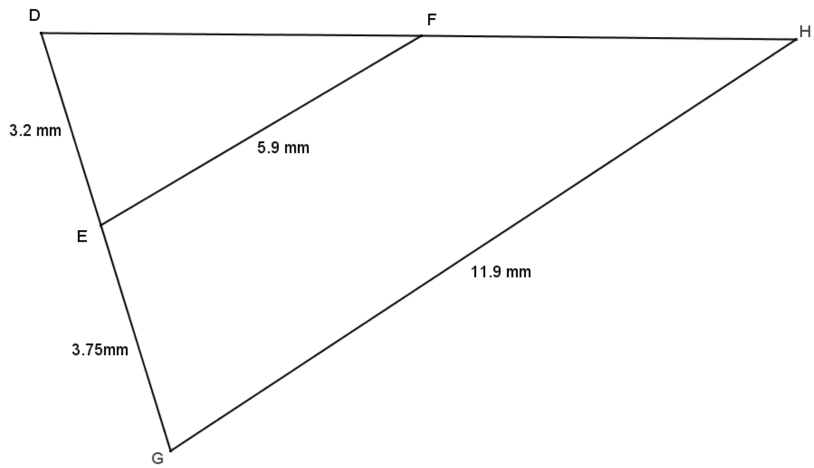
b. Use the dilation theorem to predict the length of $\overline{L'N'}$, and then measure its length directly using a ruler.

c. Does the dilation theorem appear to hold true?

3. Produce a scale drawing of $\triangle XYZ$ with point X as the center and a scale factor of $\frac{1}{4}$. Use the dilation theorem to predict $Y'Z'$, and then measure its length directly using a ruler. Does the dilation theorem appear to hold true?

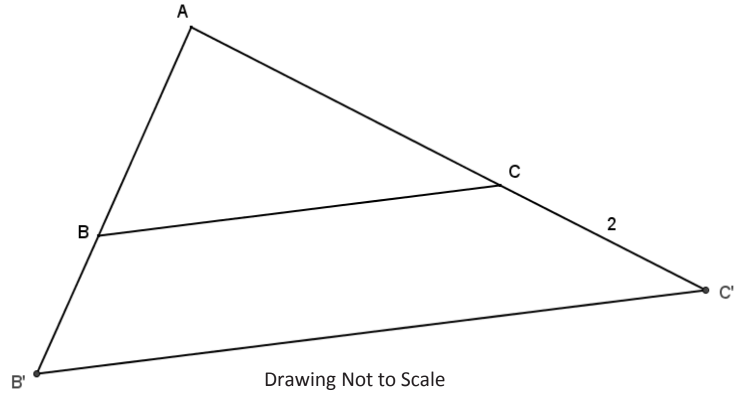


4. Given the diagram below, determine if $\triangle DEF$ is a scale drawing of $\triangle DGH$. Explain why or why not.



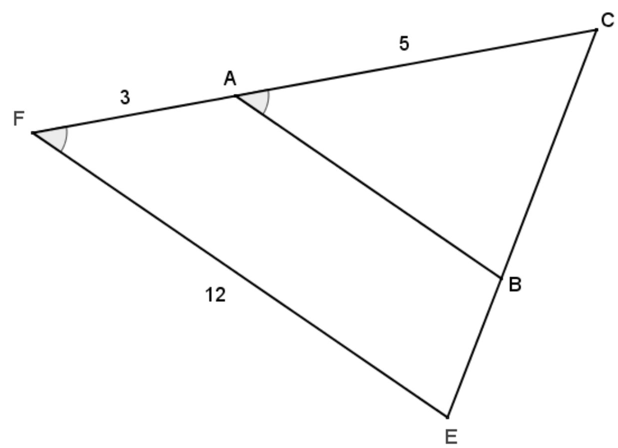
Problem Set

1. $\triangle AB'C'$ is a dilation of $\triangle ABC$ from vertex A, and $CC' = 2$. Use the given information in each part and the diagram to find $B'C'$.
   a. $AB = 9$, $AC = 4$, and $BC = 7$
   b. $AB = 4$, $AC = 9$, and $BC = 7$
   c. $AB = 7$, $AC = 9$, and $BC = 4$
   d. $AB = 7$, $AC = 4$, and $BC = 9$
   e. $AB = 4$, $AC = 7$, and $BC = 9$
   f. $AB = 9$, $AC = 7$, and $BC = 4$

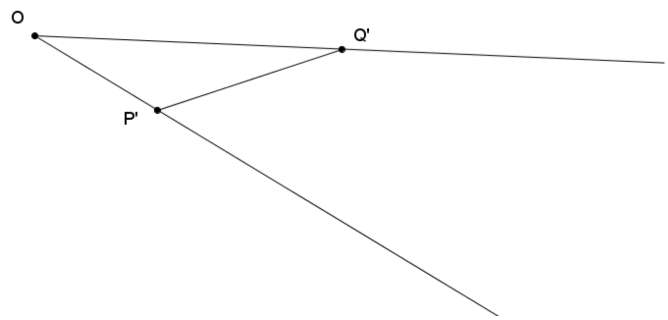


Drawing Not to Scale

2. Given the diagram, $\angle CAB \cong \angle CFE$. Find AB.



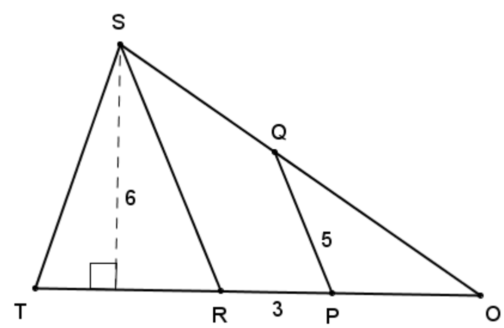
3. Use the diagram to answer each part below.



a. $\triangle OP'Q'$ is the dilated image of $\triangle OPQ$ from point O with a scale factor of $r > 1$. Draw a possible $\overline{PQ}$.

b. $\triangle OP''Q''$ is the dilated image of $\triangle OPQ$ from point O with a scale factor $k > r$. Draw a possible $\overline{P''Q''}$.

4. Given the diagram to the right, $\overline{RS} \parallel \overline{PQ}$, Area $(\triangle RST) = 15$ units^2, and Area$(\triangle OSR) = 21$ units^2, find RS.



5. Using the information given in the diagram and $UX = 9$, find Z on $\overline{XU}$ such that $\overline{YZ}$ is an altitude. Then, find YZ and XZ.