Lesson 28: Newton's Law of Cooling, Revisited

Classwork

Newton's law of cooling is used to model the temperature of an object placed in an environment of a different temperature. The temperature of the object t hours after being placed in the new environment is modeled by the formula

$$T(t) = T_a + (T_0 - T_a) \cdot e^{-kt},$$

where:

$T(t)$ is the temperature of the object after a time of t hours has elapsed,

T_a is the ambient temperature (the temperature of the surroundings), assumed to be constant and not impacted by the cooling process,

T_0 is the initial temperature of the object, and

k is the decay constant.

Mathematical Modeling Exercise 1

A crime scene investigator is called to the scene of a crime where a dead body has been found. He arrives at the scene and measures the temperature of the dead body at 9:30 p.m. to be $78.3°\text{F}$. He checks the thermostat and determines that the temperature of the room has been kept at $74°\text{F}$. At 10:30 p.m., the investigator measures the temperature of the body again. It is now $76.8°\text{F}$. He assumes that the initial temperature of the body was $98.6°\text{F}$ (normal body temperature). Using this data, the crime scene investigator proceeds to calculate the time of death. According to the data he collected, what time did the person die?

a. Can we find the time of death using only the temperature measured at 9:30 p.m.? Explain.

b. Set up a system of two equations using the data.

d. Find the value of the decay constant, k.

e. What was the time of death?

Mathematical Modeling Exercise 2

A pot of tea is heated to $90°\text{C}$. A cup of the tea is poured into a mug and taken outside where the temperature is $18°\text{C}$. After 2 minutes, the temperature of the cup of tea is approximately $65°\text{C}$.

a. Determine the value of the decay constant, k.

b. Write a function for the temperature of the tea in the mug, T, in $°\text{C}$, as a function of time, t, in minutes.

c. Graph the function T.

d. Use the graph of T to describe how the temperature decreases over time.

e. Use properties of exponents to rewrite the temperature function in the form $T(t) = 18 + 72(1 + r)^t$.

f. In Lesson 26, we saw that the value of r represents the percent change of a quantity that is changing according to an exponential function of the form $f(t) = A(1 + r)^t$. Describe what r represents in the context of the cooling tea.

g. As more time elapses, what temperature does the tea approach? Explain using both the context of the problem and the graph of the function T.

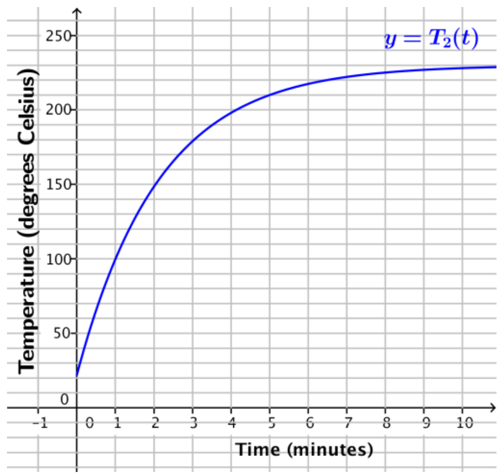
Mathematical Modeling Exercise 3

Two thermometers are sitting in a room that is $22°\text{C}$. When each thermometer reads $22°\text{C}$, the thermometers are placed in two different ovens. Data for the temperatures T_1 and T_2 of these thermometers (in °C) t minutes after being placed in the oven is provided below.

Thermometer 1:

t (minutes)	0	2	5	8	10	14
T_1 (°C)	22	75	132	173	175	176

Thermometer 2:



a. Do the table and graph given for each thermometer support the statement that Newton's law of cooling also applies when the surrounding temperature is warmer? Explain.

b. Which thermometer was placed in a hotter oven? Explain.

c. Using a generic decay constant, k, without finding its value, write an equation for each thermometer expressing the temperature as a function of time.

d. How do the equations differ when the surrounding temperature is warmer than the object rather than cooler as in previous examples?

e. How do the graphs differ when the surrounding temperature is warmer than the object rather than cooler as in previous examples?

Problem Set

1. Experiments with a covered cup of coffee show that the temperature (in degrees Fahrenheit) of the coffee can be modeled by the following equation:
$$f(t) = 112e^{-0.08t} + 68,$$
where the time is measured in minutes after the coffee was poured into the cup.
   a. What is the temperature of the coffee at the beginning of the experiment?
   b. What is the temperature of the room?
   c. After how many minutes is the temperature of the coffee $140°\text{F}$? Give your answer to 3 decimal places.
   d. What is the temperature of the coffee after a few hours have elapsed?
   e. What is the percent rate of change of the difference between the temperature of the room and the temperature of the coffee?

2. Suppose a frozen package of hamburger meat is removed from a freezer that is set at 0°F and placed in a refrigerator that is set at 38°F. Six hours after being placed in the refrigerator, the temperature of the meat is 12°F.
   a. Determine the decay constant, k.
   b. Write a function for the temperature of the meat, T in Fahrenheit, as a function of time, t in hours.
   c. Graph the function T.
   d. Describe the transformations required to graph the function T beginning with the graph of the natural exponential function $f(t) = e^t$.
   e. How long will it take the meat to thaw (reach a temperature above $32°\text{F}$)? Give answer to three decimal places.
   f. What is the percent rate of change of the difference between the temperature of the refrigerator and the temperature of the meat?

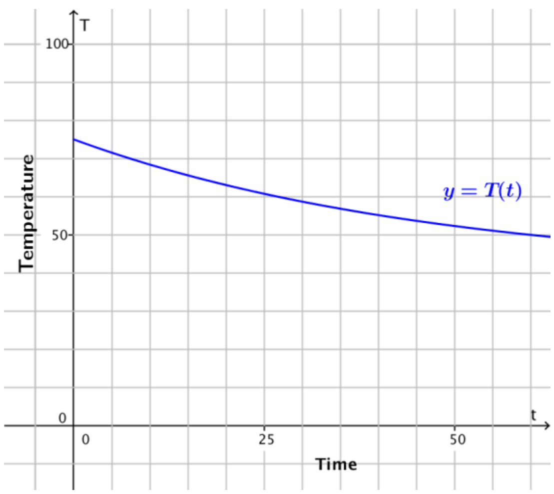
3. The table below shows the temperature of a pot of soup that was removed from the stove at time $t = 0$.

t (min)	0	10	20	30	40	50	60
T (°C)	100	34.183	22.514	20.446	20.079	20.014	20.002

   a. What is the initial temperature of the soup?
   b. What does the ambient temperature (room temperature) appear to be?
   c. Use the temperature at $t = 10$ minutes to find the decay constant, k.
   d. Confirm the value of k by using another data point from the table.
   e. Write a function for the temperature of the soup (in Celsius) as a function of time in minutes.
   f. Graph the function T.

4. Match each verbal description with its correct graph and write a possible equation expressing temperature as a function of time.
   a. A pot of liquid is heated to a boil and then placed on a counter to cool.
   b. A frozen dinner is placed in a preheated oven to cook.
   c. A can of room-temperature soda is placed in a refrigerator.

(i)



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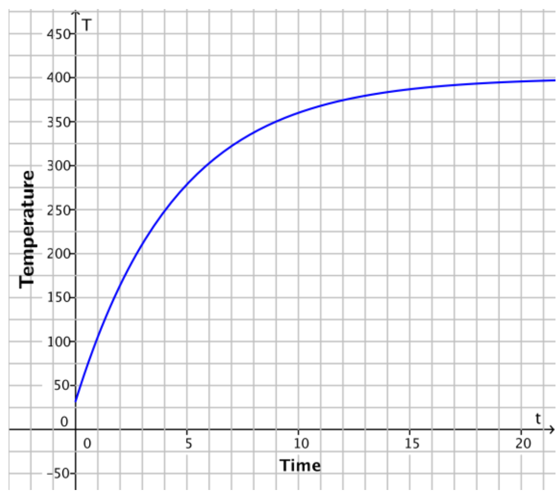

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