Lesson 21: The Graph of the Natural Logarithm Function

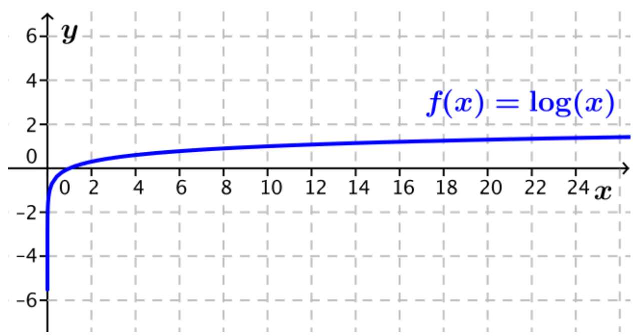
Classwork

Exploratory Challenge

Your task is to compare graphs of base b logarithm functions to the graph of the common logarithm function $f(x) = \log(x)$ and summarize your results with your group. Recall that the base of the common logarithm function is 10. A graph of f is provided below.

a. Select at least one base value from this list: $\frac{1}{10}, \frac{1}{2}, 2, 5, 20, 100$. Write a function in the form $g(x) = \log_b(x)$ for your selected base value, b.

b. Graph the functions f and g in the same viewing window using a graphing calculator or other graphing application, and then add a sketch of the graph of g to the graph of f shown below.



c. Describe how the graph of g for the base you selected compares to the graph of $f(x) = \log(x)$.

d. Share your results with your group and record observations on the graphic organizer below. Prepare a group presentation that summarizes the group's findings.

How does the graph of $g(x) = \log_b(x)$ compare to the graph of $f(x) = \log(x)$ for various values of b?	
$0 < b < 1$	
$1 < b < 10$	
$b > 10$	

Exercise 1

Use the change of base property to rewrite each logarithmic function in terms of the common logarithm function.

Base b Base 10 (Common Logarithm)

$g_1(x) = \log_{\frac{1}{4}}(x)$

$g_2(x) = \log_{\frac{1}{2}}(x)$

$g_3(x) = \log_2(x)$

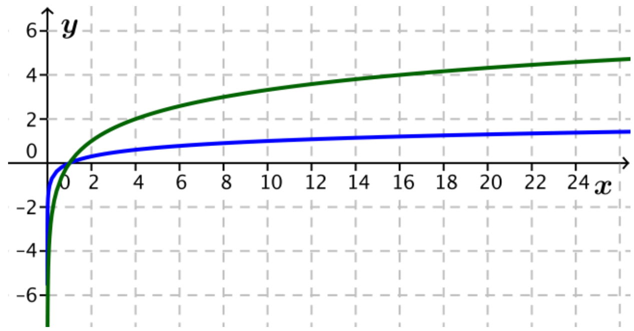
$g_4(x) = \log_5(x)$

$g_5(x) = \log_{20}(x)$

$g_6(x) = \log_{100}(x)$

Example 1: The Graph of the Natural Logarithm Function $f(x) = \ln(x)$

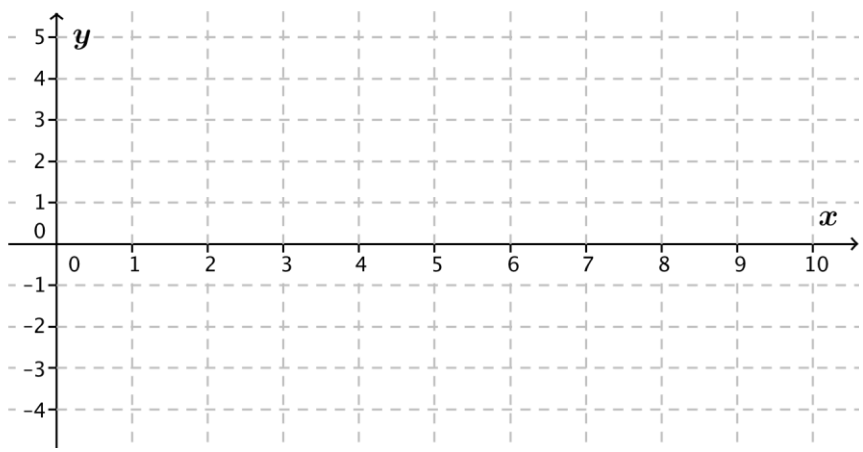
Graph the natural logarithm function below to demonstrate where it sits in relation to the graphs of the base-2 and base-10 logarithm functions.



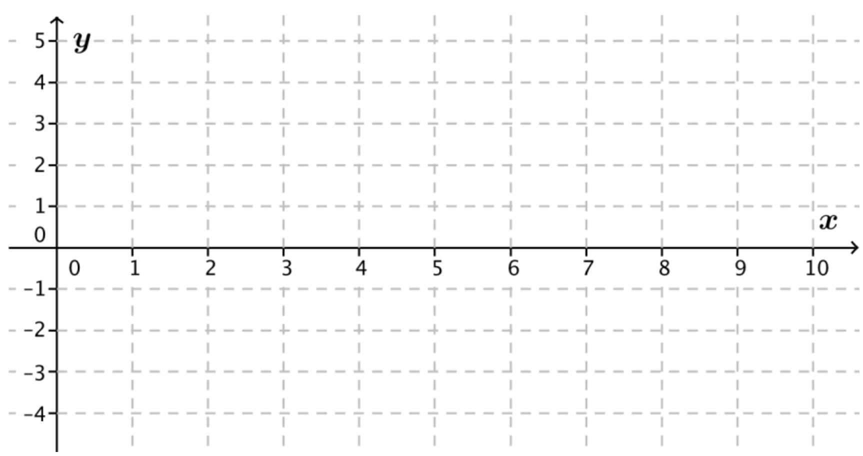
Example 2

Graph each function by applying transformations of the graphs of the natural logarithm function.

a. $f(x) = 3\ln(x-1)$



b. $g(x) = \log_6(x) - 2$



Problem Set

1. Rewrite each logarithmic function as a natural logarithm function.
   a. $f(x) = \log_5(x)$
   b. $f(x) = \log_2(x - 3)$
   c. $f(x) = \log_2\left(\frac{x}{3}\right)$
   d. $f(x) = 3 - \log(x)$
   e. $f(x) = 2\log(x + 3)$
   f. $f(x) = \log_5(25x)$

2. Describe each function as a transformation of the natural logarithm function $f(x) = \ln(x)$.
   a. $g(x) = 3\ln(x + 2)$
   b. $g(x) = -\ln(1 - x)$
   c. $g(x) = 2 + \ln(e^2 x)$
   d. $g(x) = \log_5(25x)$

3. Sketch the graphs of each function in Problem 2 and identify the key features including intercepts, decreasing or increasing intervals, and the vertical asymptote.

4. Solve the equation $1 - e^{x-1} = \ln(x)$ graphically, without using a calculator.

5. Use a graphical approach to explain why the equation $\log(x) = \ln(x)$ has only one solution.

6. Juliet tried to solve this equation as shown below using the change of base property and concluded there is no solution because $\ln(10) \neq 1$. Construct an argument to support or refute her reasoning.

$$\log(x) = \ln(x)$$
$$\frac{\ln(x)}{\ln(10)} = \ln(x)$$
$$\left(\frac{\ln(x)}{\ln(10)}\right)\frac{1}{\ln(x)} = (\ln(x))\frac{1}{\ln(x)}$$
$$\frac{1}{\ln(10)} = 1$$

7. Consider the function f given by $f(x) = \log_x(100)$ for $x > 0$ and $x \neq 1$.

   a. What are the values of $f(100)$, $f(10)$, and $f(\sqrt{10})$?

   b. Why is the value 1 excluded from the domain of this function?

   c. Find a value x so that $f(x) = 0.5$.

   d. Find a value w so that $f(w) = -1$.

   e. Sketch a graph of $y = \log_x(100)$ for $x > 0$ and $x \neq 1$.