Lesson 11: The Most Important Property of Logarithms

Classwork

Opening Exercise

Use the logarithm table below to calculate the specified logarithms.

x	$\log(x)$
1	0
2	0.3010
3	0.4771
4	0.6021
5	0.6990
6	0.7782
7	0.8451
8	0.9031
9	0.9542

a. $\log(80)$

b. $\log(7000)$

c. $\log(0.00006)$

d. $\log(3.0 \times 10^{27})$

e. $\log(9.0 \times 10^{k})$ for an integer k

Exercises 1–5

1. Use your calculator to complete the following table. Round the logarithms to four decimal places.

x	$\log(x)$
1	0
2	0.3010
3	0.4771
4	0.6021
5	0.6990
6	0.7782
7	0.8451
8	0.9031
9	0.9542

x	$\log(x)$
10	
12	
16	
18	
20	
25	
30	
36	
100	

2. Calculate the following values. Do they appear anywhere else in the table?
   a. $\log(2) + \log(4)$

   b. $\log(2) + \log(6)$

   c. $\log(3) + \log(4)$

   d. $\log(6) + \log(6)$

   e. $\log(2) + \log(18)$

   f. $\log(3) + \log(12)$

3. What pattern(s) can you see in Exercise 2 and the table from Exercise 1? Write them using logarithmic notation.

4. What pattern would you expect to find for $\log(x^2)$? Make a conjecture, and test it to see whether or not it appears to be valid.

5. Make a conjecture for a logarithm of the form $\log(xyz)$, where x, y, and z are positive real numbers. Provide evidence that your conjecture is valid.

Example 1

Use the logarithm table from Exercise 1 to approximate the following logarithms.

a. $\log(14)$

b. $\log(35)$

c. $\log(72)$

d. $\log(121)$

Exercises 6–8

6. Use your calculator to complete the following table. Round the logarithms to four decimal places.

x	$\log(x)$
2	
4	
5	
8	
10	
16	
20	
50	
100	

x	$\log(x)$
$\frac{1}{2}$	
$\frac{1}{4}$	
$\frac{1}{5}$	
$\frac{1}{8}$	
$\frac{1}{10}$	
$\frac{1}{16}$	
$\frac{1}{20}$	
$\frac{1}{50}$	
$\frac{1}{100}$	

7. What pattern(s) can you see in the table from Exercise 6? Write a conjecture using logarithmic notation.

8. Use the definition of logarithm to justify the conjecture you found in Exercise 7.

Example 2

Use the logarithm tables and the rules we have discovered to estimate the following logarithms to four decimal places.

a. $\log(2100)$

b. $\log(0.00049)$

c. $\log(42000000)$

d. $\log\left(\frac{1}{640}\right)$

Lesson Summary

- The notation $\log(x)$ is used to represent $\log_{10}(x)$.
- The most important property of base 10 logarithms is that for positive real numbers x and y,

$$\log(xy) = \log(x) + \log(y).$$

- For positive real numbers x,

$$\log\left(\frac{1}{x}\right) = -\log(x).$$

Problem Set

1. Use the table of logarithms to the right to estimate the value of the logarithms in parts (a)–(t).

x	$\log(x)$
2	0.30
3	0.48
5	0.70
7	0.85
11	1.04
13	1.11

   a. $\log(25)$
   b. $\log(27)$
   c. $\log(33)$
   d. $\log(55)$
   e. $\log(63)$
   f. $\log(75)$
   g. $\log(81)$
   h. $\log(99)$
   i. $\log(350)$
   j. $\log(0.0014)$
   k. $\log(0.077)$
   l. $\log(49000)$
   m. $\log(1.69)$
   n. $\log(6.5)$
   o. $\log\left(\frac{1}{30}\right)$
   p. $\log\left(\frac{1}{35}\right)$
   q. $\log\left(\frac{1}{40}\right)$
   r. $\log\left(\frac{1}{42}\right)$
   s. $\log\left(\frac{1}{50}\right)$
   t. $\log\left(\frac{1}{64}\right)$

2. Reduce each expression to a single logarithm of the form $\log(x)$.
   a. $\log(5) + \log(7)$
   b. $\log(3) + \log(9)$
   c. $\log(15) - \log(5)$
   d. $\log(8) + \log\left(\frac{1}{4}\right)$

3. Use properties of logarithms to write the following expressions involving logarithms of only prime numbers:
   a. $\log(2500)$
   b. $\log(0.00063)$
   c. $\log(1250)$
   d. $\log(26000000)$

4. Use properties of logarithms to show that $\log(2) - \log\left(\frac{1}{13}\right) = \log(26)$.

5. Use properties of logarithms to show that $\log(3) + \log(4) + \log(5) - \log(6) = 1$.

6. Use properties of logarithms to show that $\log\left(\frac{1}{2} - \frac{1}{3}\right) + \log(2) = -\log(3)$.

7. Use properties of logarithms to show that $\log\left(\frac{1}{3} - \frac{1}{4}\right) + \left(\log\left(\frac{1}{3}\right) - \log\left(\frac{1}{4}\right)\right) = -2\log(3)$.