Lesson 10: Building Logarithmic Tables

Classwork

Opening Exercise

Find the value of the following expressions without using a calculator.

$\mathrm{WhatPower}_{10}(1000)$ $\log_{10}(1000)$

$\mathrm{WhatPower}_{10}(100)$ $\log_{10}(100)$

$\mathrm{WhatPower}_{10}(10)$ $\log_{10}(10)$

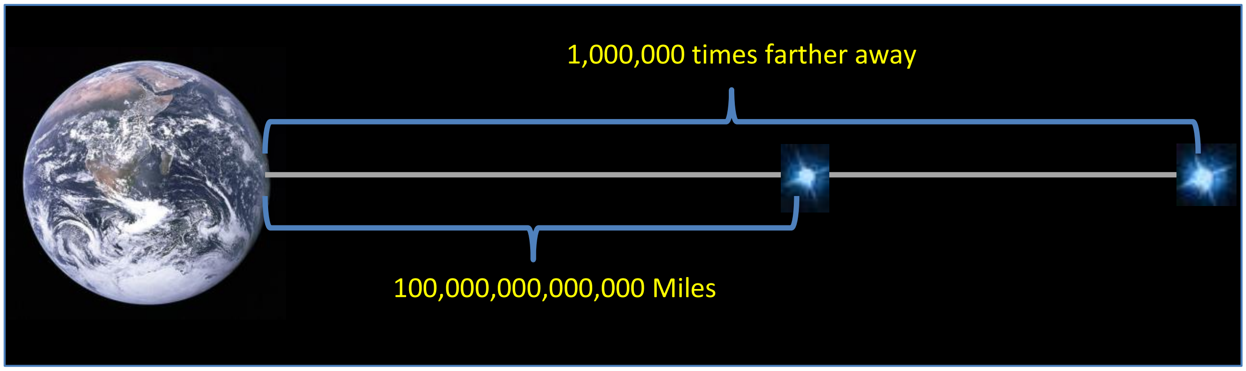
$\mathrm{WhatPower}_{10}(1)$ $\log_{10}(1)$

$\mathrm{WhatPower}_{10}\left(\frac{1}{10}\right)$ $\log_{10}\left(\frac{1}{10}\right)$

$\mathrm{WhatPower}_{10}\left(\frac{1}{100}\right)$ $\log_{10}\left(\frac{1}{100}\right)$

Formulate a rule based on your results above: If k is an integer, then $\log_{10}(10^k) =$ _______.

Example 1



Exercises

1. Find two consecutive powers of 10 so that 30 is between them. That is, find an integer exponent k so that $10^k < 30 < 10^{k+1}$.

2. From your result in Exercise 1, $\log(30)$ is between which two integers?

3. Find a number k to one decimal place so that $10^k < 30 < 10^{k+0.1}$, and use that to find under and over estimates for $\log(30)$.

4. Find a number k to two decimal places so that $10^k < 30 < 10^{k+0.01}$, and use that to find under and over estimates for $\log(30)$.

5. Repeat this process to approximate the value of $\log(30)$ to 4 decimal places.

6. Verify your result on your calculator, using the $\boxed{\text{LOG}}$ button.

7. Use your calculator to complete the following table. Round the logarithms to 4 decimal places.

x	$\log(x)$
1	
2	
3	
4	
5	
6	
7	
8	
9	

x	$\log(x)$
10	
20	
30	
40	
50	
60	
70	
80	
90	

x	$\log(x)$
100	
200	
300	
400	
500	
600	
700	
800	
900	

8. What pattern(s) can you see in the table from Exercise 7 as x is multiplied by 10? Write the pattern(s) using logarithmic notation.

9. What pattern would you expect to find for $\log(1000x)$? Make a conjecture, and test it to see whether or not it appears to be valid.

10. Use your results from Exercises 8 and 9 to make a conjecture about the value of $\log(10^k \cdot x)$ for any positive integer k.

11. Use your calculator to complete the following table. Round the logarithms to 4 decimal places.

x	$\log(x)$
1	
2	
3	
4	
5	
6	
7	
8	
9	

x	$\log(x)$
0.1	
0.2	
0.3	
0.4	
0.5	
0.6	
0.7	
0.8	
0.9	

x	$\log(x)$
0.01	
0.02	
0.03	
0.04	
0.05	
0.06	
0.07	
0.08	
0.09	

12. What pattern(s) can you see in the table from Exercise 11? Write them using logarithmic notation.

13. What pattern would you expect to find for $\log\left(\frac{x}{1000}\right)$? Make a conjecture, and test it to see whether or not it appears to be valid.

14. Combine your results from Exercises 10 and 12 to make a conjecture about the value of the logarithm for a multiple of a power of 10; that is, find a formula for $\log(10^k \cdot x)$ for any integer k.

Lesson Summary

- The notation $\log(x)$ is used to represent $\log_{10}(x)$.
- For integers k, $\log(10^k) = k$.
- For integers m and n, $\log(10^m \cdot 10^n) = \log(10^m) + \log(10^n)$.
- For integers k and positive real numbers x, $\log(10^k \cdot x) = k + \log(x)$.

Problem Set

1. Complete the following table of logarithms without using a calculator; then, answer the questions that follow.

x	$\log(x)$
$1,000,000$	
$100,000$	
$10,000$	
1000	
100	
10	

x	$\log(x)$
0.1	
0.01	
0.001	
0.0001	
0.00001	
0.000001	

   a. What is $\log(1)$? How does that follow from the definition of a base-10 logarithm?
   b. What is $\log(10^k)$ for an integer k? How does that follow from the definition of a base-10 logarithm?
   c. What happens to the value of $\log(x)$ as x gets really large?
   d. For $x > 0$, what happens to the value of $\log(x)$ as x gets really close to zero?

2. Use the table of logarithms below to estimate the values of the logarithms in parts (a)–(h).

x	$\log(x)$
2	0.3010
3	0.4771
5	0.6990
7	0.8451
11	1.0414
13	1.1139

   a. $\log(70\ 000)$
   b. $\log(0.0011)$
   c. $\log(20)$
   d. $\log(0.00005)$
   e. $\log(130\ 000)$
   f. $\log(3000)$
   g. $\log(0.07)$
   h. $\log(11000000)$

3. If $\log(n) = 0.6$, find the value of $\log(10n)$.

4. If m is a positive integer and $\log(m) \approx 3.8$, how many digits are there in m? Explain how you know.

5. If m is a positive integer and $\log(m) \approx 9.6$, how many digits are there in m? Explain how you know.

6. Vivian says $\log(452\ 000) = 5 + \log(4.52)$, while her sister Lillian says that $\log(452\ 000) = 6 + \log(0.452)$. Which sister is correct? Explain how you know.

7. Write the base-10 logarithm of each number in the form $k + \log(x)$, where k is the exponent from the scientific notation, and x is a positive real number.
   a. 2.4902×10^4
   b. 2.58×10^{13}
   c. 9.109×10^{-31}

8. For each of the following statements, write the number in scientific notation, and then write the logarithm base 10 of that number in the form $k + \log(x)$, where k is the exponent from the scientific notation, and x is a positive real number.
   a. The speed of sound is $1116\ \mathrm{ft/s}$.
   b. The distance from Earth to the sun is 93 million miles.
   c. The speed of light is $29,980,000,000\ \mathrm{cm/s}$.
   d. The weight of the earth is $5,972,000,000,000\,000,000,000,000\ \mathrm{kg}$.
   e. The diameter of the nucleus of a hydrogen atom is $0.00000000000000175\ \mathrm{m}$.
   f. For each part (a)–(e), you have written each logarithm in the form $k + \log(x)$, for integers k and positive real numbers x. Use a calculator to find the values of the expressions $\log(x)$. Why are all of these values between 0 and 1?