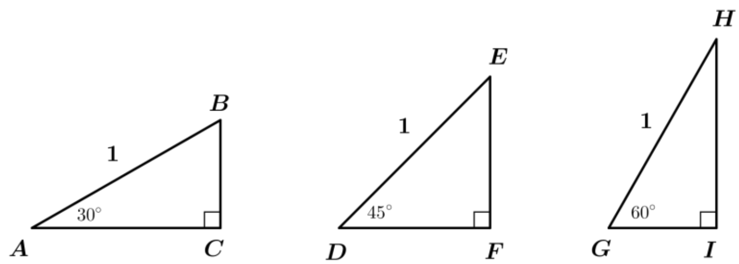
Lesson 4: From Circle-ometry to Trigonometry

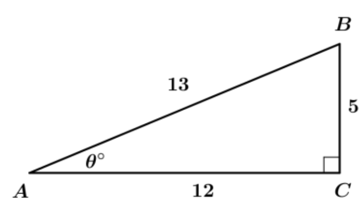
Classwork

Opening Exercises

1. Find the lengths of the sides of the right triangles below, each of which has hypotenuse of length 1.

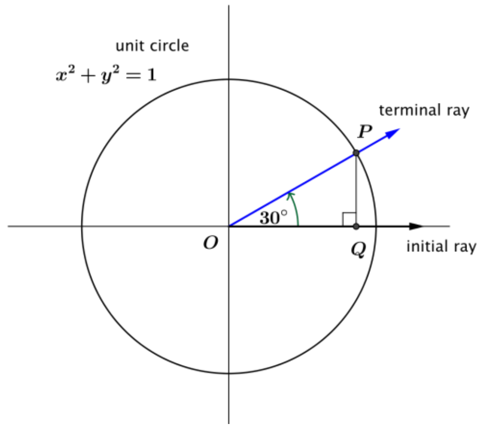


2. Given the following right triangle $\triangle ABC$ with $m\angle A = \theta°$, find $\sin(\theta°)$ and $\cos(\theta°)$.



Example 1

Suppose that point P is the point on the unit circle obtained by rotating the initial ray through $30°$. Find $\sin(30°)$ and $\cos(30°)$.



What is the length OQ of the horizontal leg of our triangle?

What is the length QP of the vertical leg of our triangle?

What is $\sin(30°)$?

What is $\cos(30°)$?

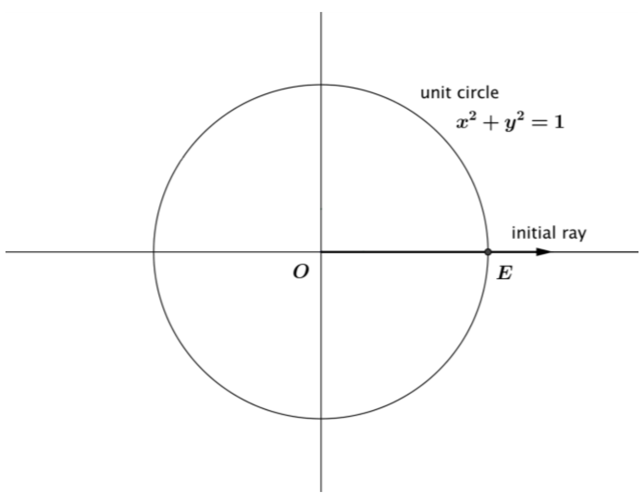
Exercises 1–2

1. Suppose that P is the point on the unit circle obtained by rotating the initial ray through $45°$. Find $\sin(45°)$ and $\cos(45°)$.

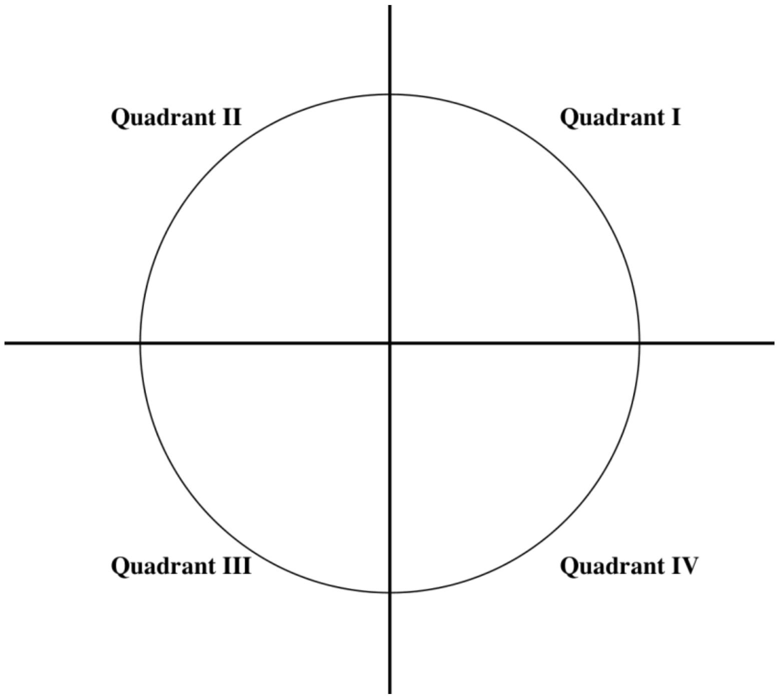
2. Suppose that P is the point on the unit circle obtained by rotating the initial ray through $60°$. Find $\sin(60°)$ and $\cos(60°)$.

Example 2

Suppose that P is the point on the unit circle obtained by rotating the initial ray through $150°$. Find $\sin(150°)$ and $\cos(150°)$.



Discussion



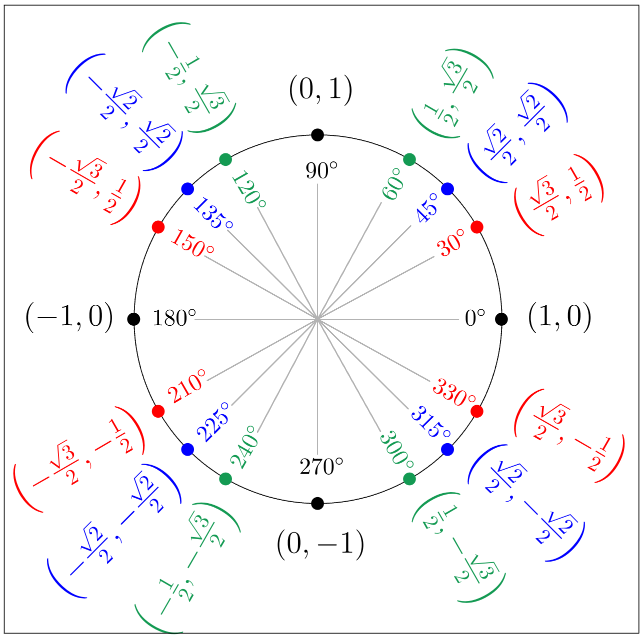
Exercises 3–5

3. Suppose that P is the point on the unit circle obtained by rotating the initial ray counterclockwise through 120 degrees. Find the measure of the reference angle for $120°$, and then find $\sin(120°)$ and $\cos(120°)$.

4. Suppose that P is the point on the unit circle obtained by rotating the initial ray counterclockwise through $240°$. Find the measure of the reference angle for $240°$, and then find $\sin(240°)$ and $\cos(240°)$.

5. Suppose that P is the point on the unit circle obtained by rotating the initial ray counterclockwise through 330 degrees. Find the measure of the reference angle for $330°$, and then find $\sin(330°)$ and $\cos(330°)$.

Discussion



Lesson Summary

In this lesson we formalized the idea of the height and co-height of a Ferris wheel and defined the sine and cosine functions that give the x- and y- coordinates of the intersection of the unit circle and the initial ray rotated through θ degrees, for most values of θ with $0 < \theta < 360$.

- The value of $\cos(\theta°)$ is the x-coordinate of the intersection point of the terminal ray and the unit circle.
- The value of $\sin(\theta°)$ is the y-coordinate of the intersection point of the terminal ray and the unit circle.
- The sine and cosine functions have domain of all real numbers and range $[-1,1]$.

Problem Set

1. Fill in the chart. Write in the reference angles and the values of the sine and cosine functions for the indicated values of θ.

Amount of rotation, $\boldsymbol{\theta}$, in degrees	**Measure of Reference Angle, in degrees**	$\mathbf{cos}(\boldsymbol{\theta}°)$	$\mathbf{sin}(\boldsymbol{\theta}°)$
120			
135			
150			
225			
240			
300			
330			

2. Using geometry, Jennifer correctly calculated that $\sin(15°) = \frac{1}{2}\sqrt{2-\sqrt{3}}$. Based on this information, fill in the chart.

Amount of rotation, $\boldsymbol{\theta}$, in degrees	**Measure of Reference Angle, in degrees**	$\boldsymbol{\cos(\theta°)}$	$\boldsymbol{\sin(\theta°)}$
15			
165			
195			
345			

3. Suppose $0 < \theta < 90$ and $\sin(\theta°) = \frac{1}{\sqrt{3}}$. What is the value of $\cos(\theta°)$?

4. Suppose $90 < \theta < 180$ and $\sin(\theta°) = \frac{1}{\sqrt{3}}$. What is the value of $\cos(\theta°)$?

5. If $\cos(\theta°) = -\frac{1}{\sqrt{5}}$, what are two possible values of $\sin(\theta°)$?

6. Johnny rotated the initial ray through θ degrees, found the intersection of the terminal ray with the unit circle, and calculated that $\sin(\theta°) = \sqrt{2}$. Ernesto insists that Johnny made a mistake in his calculation. Explain why Ernesto is correct.

7. If $\sin(\theta°) = 0.5$, and we know that $\cos(\theta°) < 0$, then what is the smallest possible positive value of θ?

8. The vertices of triangle $\triangle ABC$ have coordinates $A(0,0)$, $B(12,5)$, and $C(12,0)$.
   a. Argue that $\triangle ABC$ is a right triangle.
   b. What are the coordinates where the hypotenuse of $\triangle ABC$ intersects the unit circle $x^2 + y^2 = 1$?
   c. Let θ denote the number of degrees of rotation from $\overrightarrow{AC}$ to $\overrightarrow{AB}$. Calculate $\sin(\theta°)$ and $\cos(\theta°)$.

9. The vertices of triangle $\triangle ABC$ have coordinates $A(0,0)$, $B(4,3)$, and $C(4,0)$. The vertices of triangle $\triangle ADE$ are $A(0,0)$, $D(3,4)$, and $E(3,0)$.
   a. Argue that $\triangle ABC$ is a right triangle.
   b. What are the coordinates where the hypotenuse of $\triangle ABC$ intersects the unit circle $x^2 + y^2 = 1$?
   c. Let θ denote the number of degrees of rotation from $\overrightarrow{AC}$ to $\overrightarrow{AB}$. Calculate $\sin(\theta°)$ and $\cos(\theta°)$.
   d. Argue that $\triangle ADE$ is a right triangle.
   e. What are the coordinates where the hypotenuse of $\triangle ADE$ intersects the unit circle $x^2 + y^2 = 1$?
   f. Let ϕ denote the number of degrees of rotation from $\overrightarrow{AE}$ to $\overrightarrow{AD}$. Calculate $\sin(\phi°)$ and $\cos(\phi°)$.
   g. What is the relation between the sine and cosine of θ and the sine and cosine of ϕ?

10. Use a diagram to explain why $\sin(135°) = \sin(45°)$, but $\cos(135°) \neq \cos(45°)$.