## Lesson 21: Modeling Riverbeds with Polynomials

## Classwork

## Mathematical Modeling Exercise

The Environmental Protection Agency (EPA) is studying the flow of a river in order to establish flood zones. The EPA hired a surveying company to determine the flow rate of the river, measured as volume of water per minute. The firm set up a coordinate system and found the depths of the river at five locations as shown on the graph below. After studying the data, the firm decided to model the riverbed with a polynomial function and divide the area into six regions that are either trapezoidal or triangular so that the overall area can be easily estimated. The firm needs to approximate the depth of the river at two more data points in order to do this.


1. Find a polynomial $P$ that fits the five given data points.
2. Use the polynomial to estimate the depth of the river at $x=40$ and $x=80$.
3. Estimate the area of the cross section.

Suppose that the river flow speed was measured to be an average speed of $176 \frac{\mathrm{ft} .}{\mathrm{min}}$ the cross section.
4. What is the volumetric flow of the water (the volume of water per minute)?
5. Convert the flow to gallons per minute. [Note: 1 cubic foot $\approx 7.48052$ gallons.]

## Problem Set

1. As the leader of the surveying team, write a short report to the EPA on your findings from the in-class exercises. Be sure to include data and calculations.
2. Suppose that depths of the riverbed were measured for a different cross-section of the river.
a. Use Wolfram Alpha to find the interpolating polynomial $Q$ with values:

$$
\begin{array}{llll}
Q(0)=0, & Q(16.5)=-27.4, & Q(44.4)=-19.6, & Q(77.6)=-25.1, \\
Q(123.3)=-15.0, & Q(131.1)=-15.1, & Q(150)=0 .
\end{array}
$$

b. Sketch the cross-section of the river, and estimate its area.
c. Suppose that the speed of the water was measured at $124 \frac{\mathrm{ft}}{\mathrm{min}}$. What is the approximate volumetric flow in this section of the river, measured in gallons per minute?

