## Lesson 20: Modeling Riverbeds with Polynomials

## Classwork

## Mathematical Modeling Exercise

The Environmental Protection Agency (EPA) is studying the flow of a river in order to establish flood zones. The EPA hired a surveying company to determine the flow rate of the river, measured as volume of water per minute. The firm set up a coordinate system and found the depths of the river at five locations as shown on the graph below. After studying the data, the firm decided to model the riverbed with a polynomial function and divide the cross-sectional area into six regions that are either trapezoidal or triangular so that the overall area can be easily estimated. The firm needs to approximate the depth of the river at two more data points in order to do this.


Draw the four trapezoids and two triangles that will be used to estimate the cross-sectional area of the riverbed.

## Example 1

Find a polynomial $P$ such that $P(0)=28, P(2)=0$, and $P(8)=12$.


## Example 2

Find a degree 3 polynomial $P$ such that $P(-1)=-3, P(0)=-2, P(1)=-1$, and $P(2)=6$.

| Function <br> Value | Substitute the data point into the <br> current form of the equation for $\boldsymbol{P}$. | Apply the remainder <br> theorem to $\boldsymbol{a}, \boldsymbol{b}$, and $\boldsymbol{c}$ | Rewrite the equation for $\boldsymbol{P}$ in <br> terms of $\boldsymbol{a}, \boldsymbol{b}$, or $\boldsymbol{c}$. |
| :---: | :---: | :---: | :---: |
| $P(-1)=-3$ |  |  |  |
| $P(0)=-2$ |  |  |  |
| $P(1)=-1$ |  |  |  |
| $P(2)=6$ |  |  |  |

## Lesson Summary

A linear polynomial is determined by 2 points on its graph.
A degree 2 polynomial is determined by 3 points on its graph.
A degree 3 polynomial is determined by 4 points on its graph.
A degree 4 polynomial is determined by 5 points on its graph.
The remainder theorem can be used to find a polynomial $P$ whose graph will pass through a given set of points.

## Problem Set

1. Suppose a polynomial function $P$ is such that $P(2)=5$ and $P(3)=12$.
a. What is the largest-degree polynomial that can be uniquely determined given the information?
b. Is this the only polynomial that satisfies $P(2)=5$ and $P(3)=12$ ?
c. Use the remainder theorem to find the polynomial $P$ of least degree that satisfies the two points given.
d. Verify that your equation is correct by demonstrating that it satisfies the given points.
2. Write a quadratic function $P$ such that $P(0)=-10, P(5)=0$, and $P(7)=18$ using the specified method.
a. Setting up a system of equations
b. Using the remainder theorem
3. Find a degree-three polynomial function $P$ such that $P(-1)=0, P(0)=2, P(2)=12$, and $P(3)=32$. Use the table below to organize your work. Write your answer in standard form, and verify by showing that each point satisfies the equation.

| Function <br> Value | Substitute the data point into <br> the current form of the <br> equation for $\boldsymbol{P}$. | Apply the remainder theorem <br> to $\boldsymbol{a}, \boldsymbol{b}$, and $\boldsymbol{c}$. | Rewrite the equation for $\boldsymbol{P}$ in <br> terms of $\boldsymbol{a}, \boldsymbol{b}$, or $\boldsymbol{c}$. |
| :--- | :--- | :--- | :--- |
| $P(-1)=0$ |  |  |  |
| $P(0)=2$ |  |  |  |
| $P(2)=12$ |  |  |  |
| $P(3)=32$ |  |  |  |

4. The method used in Problem 3 is based on the Lagrange interpolation method. Research Joseph-Louis Lagrange, and write a paragraph about his mathematical work.
