Lesson 19: The Remainder Theorem

Classwork

Exercises 1–3

1. Consider the polynomial function $f(x) = 3x^2 + 8x - 4$.

   a. Divide f by $x - 2$.

   b. Find $f(2)$.

2. Consider the polynomial function $g(x) = x^3 - 3x^2 + 6x + 8$.

   a. Divide g by $x + 1$.

   b. Find $g(-1)$.

3. Consider the polynomial function $h(x) = x^3 + 2x - 3$.

   a. Divide h by $x - 3$.

   b. Find $h(3)$.

Exercises 4–6

4. Consider the polynomial $P(x) = x^3 + kx^2 + x + 6$.
   a. Find the value of k so that $x + 1$ is a factor of P.

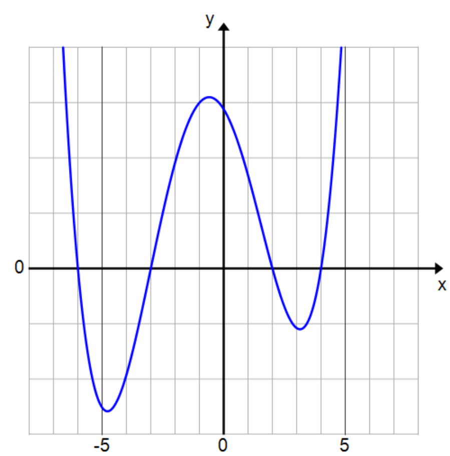
   b. Find the other two factors of P for the value of k found in part (a).

5. Consider the polynomial $P(x) = x^4 + 3x^3 - 28x^2 - 36x + 144$.
   a. Is 1 a zero of the polynomial P?

   b. Is $x + 3$ one of the factors of P?

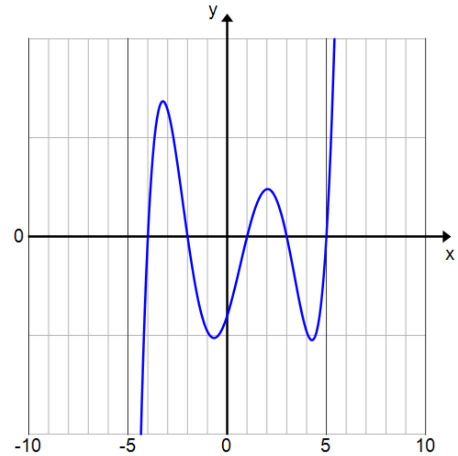
   c. The graph of P is shown to the right. What are the zeros of P?

   d. Write the equation of P in factored form.



6. Consider the graph of a degree 5 polynomial shown to the right, with x-intercepts -4, -2, 1, 3, and 5.

   a. Write a formula for a possible polynomial function that the graph represents using c as the constant factor.



   b. Suppose the y-intercept is -4. Find the value of c so that the graph of P has y-intercept -4.

Lesson Summary

REMAINDER THEOREM: Let P be a polynomial function in x, and let a be any real number. Then there exists a unique polynomial function q such that the equation

$$P(x) = q(x)(x - a) + P(a)$$

is true for all x. That is, when a polynomial is divided by $(x - a)$, the remainder is the value of the polynomial evaluated at a.

FACTOR THEOREM: Let P be a polynomial function in x, and let a be any real number. If a is a zero of P, then $(x - a)$ is a factor of P.

Example: If $P(x) = x^2 - 3$ and $a = 4$, then $P(x) = (x + 4)(x - 4) + 13$ where $q(x) = x + 4$ and $P(4) = 13$.

Example: If $P(x) = x^3 - 5x^2 + 3x + 9$, then $P(3) = 27 - 45 + 9 + 9 = 0$, so $(x - 3)$ is a factor of P.

Problem Set

1. Use the remainder theorem to find the remainder for each of the following divisions.
   a. $\frac{(x^2+3x+1)}{(x+2)}$
   b. $\frac{x^3-6x^2-7x+9}{(x-3)}$
   c. $\frac{32x^4+24x^3-12x^2+2x+1}{(x+1)}$
   d. $\frac{32x^4+24x^3-12x^2+2x+1}{(2x-1)}$,

   Hint for part (d): Can you rewrite the division expression so that the divisor is in the form $(x - c)$ for some constant c?

2. Consider the polynomial $P(x) = x^3 + 6x^2 - 8x - 1$. Find $P(4)$ in two ways.

3. Consider the polynomial function $P(x) = 2x^4 + 3x^2 + 12$.
   a. Divide P by $x + 2$, and rewrite P in the form (divisor)(quotient) + remainder.
   b. Find $P(-2)$.

4. Consider the polynomial function $P(x) = x^3 + 42$.
   a. Divide P by $x - 4$, and rewrite P in the form (divisor)(quotient)+remainder.
   b. Find $P(4)$.

5. Explain why for a polynomial function P, $P(a)$ is equal to the remainder of the quotient of P and $x - a$.

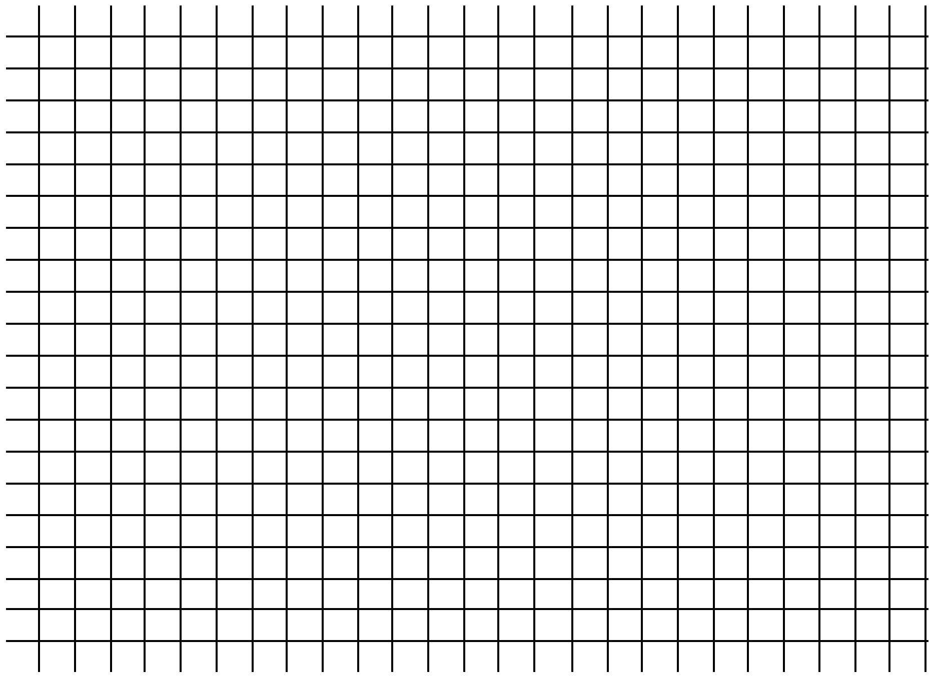
6. Is $x - 5$ a factor of the function $f(x) = x^3 + x^2 - 27x - 15$? Show work supporting your answer.

7. Is $x + 1$ a factor of the function $f(x) = 2x^5 - 4x^4 + 9x^3 - x + 13$? Show work supporting your answer.

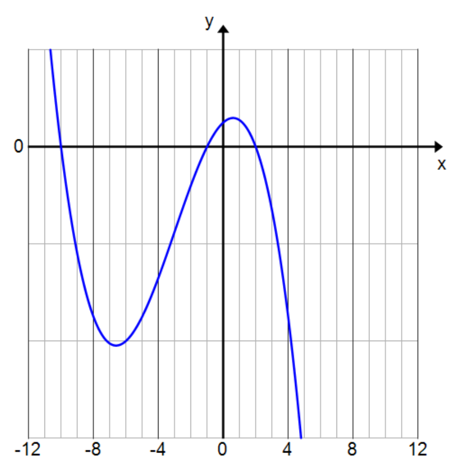
8. A polynomial function p has zeros of $2, 2, -3, -3, -3$, and 4. Find a possible formula for P, and state its degree. Why is the degree of the polynomial not 3?

9. Consider the polynomial function $P(x) = x^3 - 8x^2 - 29x + 180$.
   a. Verify that $P(9) = 0$. Since $P(9) = 0$, what must one of the factors of P be?
   b. Find the remaining two factors of P.
   c. State the zeros of P.
   d. Sketch the graph of P.

10. Consider the polynomial function $P(x) = 2x^3 + 3x^2 - 2x - 3$.
   a. Verify that $P(-1) = 0$. Since $P(-1) = 0$, what must one of the factors of P be?
   b. Find the remaining two factors of P.
   c. State the zeros of P.
   d. Sketch the graph of P.



11. The graph to the right is of a third-degree polynomial function f.
   a. State the zeros of f.
   b. Write a formula for f in factored form using c for the constant factor.
   c. Use the fact that $f(-4) = -54$ to find the constant factor c.
   d. Verify your equation by using the fact that $f(1) = 11$.



12. Find the value of k so that $\frac{x^3-kx^2+2}{x-1}$ has remainder 8.

13. Find the value k so that $\frac{kx^3+x-k}{x+2}$ has remainder 16.

14. Show that $x^{51} - 21x + 20$ is divisible by $x - 1$.

15. Show that $x + 1$ is a factor of $19x^{42} + 18x - 1$.

Write a polynomial function that meets the stated conditions.

16. The zeros are -2 and 1.

17. The zeros are -1, 2, and 7.

18. The zeros are $-\frac{1}{2}$ and $\frac{3}{4}$.

19. The zeros are $-\frac{2}{3}$ and 5, and the constant term of the polynomial is -10.

20. The zeros are 2 and $-\frac{3}{2}$, the polynomial has degree 3, and there are no other zeros.