Lesson 13: Mastering Factoring

Classwork

Opening Exercises

Factor each of the following expressions. What similarities do you notice between the examples in the left column and those on the right?

a. $x^2 - 1$

b. $9x^2 - 1$

c. $x^2 + 8x + 15$

d. $4x^2 + 16x + 15$

e. $x^2 - y^2$

f. $x^4 - y^4$

Example 1

Write $9 - 16x^4$ as the product of two factors.

Example 2

Factor $4x^2y^4 - 25x^4z^6$.

Exercise 1

1. Factor the following expressions:
   a. $4x^2 + 4x - 63$

   b. $12y^2 - 24y - 15$

Exercises 2–4

Factor each of the following, and show that the factored form is equivalent to the original expression.

2. $a^3 + 27$

3. $x^3 - 64$

4. $2x^3 + 128$

Lesson Summary

In this lesson we learned additional strategies for factoring polynomials.

- The difference of squares identity $a^2 - b^2 = (a - b)(a + b)$ can be used to factor more advanced binomials.
- Trinomials can often be factored by looking for structure and then applying our previous factoring methods.
- Sums and differences of cubes can be factored by the formulas

$$x^3 + a^3 = (x + a)(x^2 - ax - a^2)$$
$$x^3 - a^3 = (x - a)(x^2 + ax + a^2).$$

Problem Set

1. If possible, factor the following expressions using the techniques discussed in this lesson.

   a. $25x^2 - 25x - 14$
   b. $9x^2y^2 - 18xy + 8$
   c. $45y^2 + 15y - 10$
   d. $y^6 - y^3 - 6$
   e. $x^3 - 125$
   f. $2x^4 - 16x$
   g. $9x^2 - 25y^4z^6$
   h. $36x^6y^4z^2 - 25x^2z^{10}$
   i. $4x^2 + 9$
   j. $x^4 - 36$
   k. $1 + 27x^9$
   l. $x^3y^6 + 8z^3$

2. Consider the polynomial expression $y^4 + 4y^2 + 16$.

   a. Is $y^4 + 4y^2 + 16$ factorable using the methods we have seen so far?
   b. Factor $y^6 - 64$ first as a difference of cubes, and then factor completely: $(y^2)^3 - 4^3$.
   c. Factor $y^6 - 64$ first as a difference of squares, and then factor completely: $(y^3)^2 - 8^2$.
   d. Explain how your answers to parts (b) and (c) provide a factorization of $y^4 + 4y^2 + 16$.
   e. If a polynomial can be factored as either a difference of squares or a difference of cubes, which formula should you apply first, and why?

3. Create expressions that have a structure that allows them to be factored using the specified identity. Be creative, and produce challenging problems!

   a. Difference of squares
   b. Difference of cubes
   c. Sum of cubes